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Calibration of higher eigenmode spring constants of atomic force microscope cantilevers

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Abstract
Standard spring constant calibration methods are compared when applied to higher eigenmodes of cantilevers used in dynamic atomic force microscopy (dAFM). Analysis shows that Sader's original method (Sader et al 1999 Rev. Sci. Instrum. 70 3967–9), which relies on a priori knowledge of the eigenmode shape, is poorly suited for the calibration of higher eigenmodes. On the other hand, the thermal noise method (Hutter and Bechhoefer 1993 Rev. Sci. Instrum. 64 1868–73) does not require knowledge of the eigenmode and remains valid for higher eigenmodes of the dAFM probe. Experimental measurements of thermal vibrations in air for three representative cantilevers are provided to support the theoretical results.

1. Introduction

Atomic force microscopy (AFM) stands out among other microscopy techniques for its capability to render images of heterogeneous surfaces with nanometric resolution [1–3]. In recent years several attempts have been proposed to also convert AFM into an analytical tool, able to measure quantitatively and precisely the local properties. The problem of force inversion (i.e. to extract the actual value of the interaction between tip and sample from experimental observables) has been solved under different approximations in amplitude-[4–7] and frequency-modulation [8, 9] modes.

To combine both quantitative material information and high resolution imaging a number of groups have opted for the excitation and/or detection of higher harmonics [10] of the cantilever motion and/or higher eigenmodes [11] which are related to other flexural [12] or torsional [13] resonances of the probe. However, the quest for quantitative information in AFM greatly depends on accurate calibration of the relevant spring constant(s) for the application. Quasi-static AFM modes require the static spring constant, i.e. the constant relating a point-load applied at the tip and the resulting deflection of the tip, to be calibrated [14]. Dynamic AFM modes require the equivalent spring constants of the participating eigenmodes to be calibrated. However, the close similarity between the spring constant of the fundamental eigenmode and the static spring constant [15] has resulted in the two rarely being distinguished from one another in the literature. The evolution of dynamic AFM to incorporate higher eigenmodes requires that distinction is made between the static spring constant, \( k_{\text{static}} \), and the equivalent spring constant of higher eigenmodes, \( k_n \) \((n > 1)\). Moreover, we must reconsider calibration methods, which were originally developed specifically to calibrate \( k_{\text{static}} \), and how they can be modified to produce \( k_n \) [16].

Several methods have been suggested to calibrate \( k_{\text{static}} \) [17–31], although two of them have been particularly popular [32]. The first was proposed by Sader [19]. Remarkably, this method uses the predicted hydrodynamic damping affecting a rectangular AFM probe immersed in a fluid to calibrate \( k_{\text{static}} \) in terms of the measured quality factor and resonance frequency, as well as the geometrical dimensions of the beam. On the other hand the thermal noise method, originally proposed by Hutter and Bechhoefer [17] but later modified by Butt and Jaschke [16], takes advantage of the equipartition theorem which relates the thermal fluctuations
of the cantilever to the equilibrium temperature of the surrounding fluid through $k_{\text{static}}$. While these methods are mostly used for calibration of the fundamental eigenmode, it remains unclear whether or not these methods can be extended to higher eigenmodes.

In this paper, we investigate the calibration of higher eigenmodes through adaptations of the Sader method and the thermal noise method. We find considerable limitations to the calibration of higher eigenmodes due to the necessity of a priori knowledge of the eigenmode. Specifically, the sensitivity of higher eigenmodes to the tip mass and the cantilever geometry introduces large errors in the Sader method when applied to higher eigenmodes [33]. We find that thermal calibration is much more amenable to the calibration of higher eigenmodes, as long as the resonance frequencies, $f_n$, are adequately spaced. Specifically, the spacing of the resonances, $f_2 - f_1$, should be greater than their bandwidths, $f_n/Q_n$, with $Q_n$ the eigenmode quality factor. Experimental results are presented for three representative cantilevers: a tipped rectangular cantilever, a tipless rectangular cantilever, and a tipless picket-shaped cantilever.

2. Theory

We begin with the application of standard beam theory to demonstrate the influence of a particle mass at the free end of a uniform rectangular cantilever in higher eigenmode spring constants. The Euler–Bernoulli partial differential equation (PDE) describes the motion in time of a one-dimensional beam in the presence of external forces, among which we will only consider the hydrodynamic loading of the surrounding fluid. In the Fourier domain it is expressed as [34],

$$\frac{EI}{L^4}W_{xxx}(x, \omega) - \rho b h \omega^2 W(x, \omega) = \frac{\pi}{4} \rho f b^2 \omega^2 \Gamma(\omega) W(x, 0)$$

where $W(x, \omega)$ is the Fourier transform of the out-of-plane displacement of a beam element, placed in its longitudinal normalized coordinate, $x$. $E$ is the Young’s modulus, $I$ is the moment of inertia and $L$ the length of the cantilever. The angular frequency is referred to as $\omega$, while partial derivatives with respect to $x$ are denoted by $(\cdot)_x$. The cantilever density, width and thickness are denoted respectively by $\rho$, $b$ and $h$. The right-hand side of equation (1) represents the hydrodynamic loading, with $\rho_f$ and $\Gamma(\omega)$, respectively, the surrounding fluid density and the (complex) hydrodynamic function of the problem, as defined in [34].

We introduce the tip effect approximated as a particle mass, $m_{\text{tip}}$, which leads to a shear force balance at the free end of the cantilever. The boundary conditions of a beam clamped at $x = 0$, and supporting a mass particle at $x = 1$ are given by [33, 35, 36]

$$W(0, \omega) = W(0, 0) = W_{xx}(1, \omega) = 0; \frac{EI}{L^4}W_{xxx}(1, \omega) = -m_{\text{tip}}\omega^2 W(1, \omega).$$

The eigenmodes, $\phi_n(x)$, and the characteristic equation for the eigenvalues, $\alpha_n$, follow from separation of variables in the Euler–Bernoulli PDE, equation (1), and imposition of the boundary conditions, equations (2). Because the damping in air is small, the undamped eigenmodes and the damped eigenmodes are nearly identical [37]. Therefore, the imaginary part of the hydrodynamic function can be neglected in equation (1), yielding

$$\Phi_n(x; \alpha) = \cos(\alpha_n x) - \cos(\alpha_n x)$$

$$= \frac{\cos(\alpha_n) + \cos(\alpha_n)(\sin(\alpha_n) - \sinh(\alpha_n))}{\sin(\alpha_n) + \sinh(\alpha_n)}$$

where $\alpha_n$ is a solution of

$$1 + \cos \alpha_n \cosh \alpha_n + m* \alpha_n (\sin \alpha_n \cos \alpha_n - \sin \alpha_n \cosh \alpha_n) = 0$$

where $m* \equiv m_{\text{tip}}/(m_{\text{beam}} + m_{\text{hydro}})$ is the reduced or dimensionless tip mass, and with $m_{\text{beam}} \equiv \rho b h L$ and $m_{\text{hydro}} \equiv \pi \rho_f b^2 L \Re[\Gamma(\omega)]/4$ the cantilever and hydrodynamic added masses, respectively. Here, the real part of the hydrodynamic function is denoted by $\Re[\Gamma]$. Equations (3) and (4) show that the shapes of the eigenmodes, $\phi_n(x; m*)$, are determined by their eigenvalues, $\alpha_n$, which in turn are determined by the value of $m*$. We have omitted the dependence of $\alpha_n$ on $m$, $\alpha(m*)$, for simplicity of notation. At this point we would like to remark that the value of $m*$ depends on the eigenmode considered because the hydrodynamic added mass, $m_{\text{hydro}}$, is frequency dependent, according to Sader’s theory. However, for cantilevers immersed in air, $m_{\text{hydro}}$ is negligible, which leads to $m* = m_{\text{tip}}/m_{\text{beam}}$, regardless of the eigenmode number. We will assume this situation during the present work.

Here we are concerned with the measurement of the equivalent spring constant related to the tip deflection of the $n$th eigenmode, $k_n$. Accordingly, we apply the energy equivalence principle [15], relating $k_n$ to $\Phi_n$ (the prime denotes total derivative with respect to $x$),

$$k_n(m*) = \frac{EI \int_0^1 dx [\Phi_n'(x; m*)]^2}{L^4 [\Phi_n(1; m*)]^2}.$$

From the above results we deduce that when the eigenmode shapes are modified by $m*$, the equivalent spring constants will be affected too. On the other hand, if $m* = 0$ we recover the eigenmode shapes and equivalent spring constants corresponding to rectangular tipless cantilevers [15].

Figure 1(a) shows the dependence of the ratio $k_n/k_{\text{static}}$ for eigenmodes $n = 1, 2$ with respect to the reduced mass, $m*$, with $k_{\text{static}} \equiv 3EI/L^3$ (constant) the cantilever static spring constant [38]. While $k_1/k_{\text{static}}$ is found to slowly decrease from 1.03 to 1.00 when varying $m*$ between 0 and 0.2, $k_2/k_{\text{static}}$ dramatically increases from 40.2 to 125. Thus, the presence of a tip mass at the free end of the cantilever does not modify the first eigenmode spring constant significantly, but causes major changes in $k_2$. For a tip mass that is 10% of the cantilever mass, $m* = 0.1$, the equivalent spring constant will be $k_2 = 74.9k_{\text{static}}$. The increase in $k_2$ with respect to $m*$ occurs primarily because $\phi_2(1; m*)$ becomes small as $\Phi_2(x; m*)$ begins to resemble the fundamental eigenmode of a clamped–pinned beam for large values of $m*$. On the other hand, $\phi_1(x; m*)$ will approach the profile of a cantilever with a static point-load applied at the free end, and thus $k_1 \rightarrow k_{\text{static}}$. 


Once we have calculated the eigenmodes of the problem, \( \Phi_n(x; m^*) \), we can implement the general form of the Sader formula to calculate the fundamental and higher eigenmodes equivalent spring constant, \( n \geq 1 \),

\[
    k_n \text{Sader} = \mu_n \frac{\pi}{4} L b^2 \rho / \text{Im}[\Gamma(\omega_n)] \dot{\omega}_n^2 Q_n
\]

where \( \omega_n \) and \( Q_n \) are, respectively, the measured resonance frequency and quality factor of the eigenmode considered. The imaginary part of the hydrodynamic function is denoted by \( \text{Im}[\Gamma] \). The effective mass factor of the beam, \( \mu_n \), is defined as

\[
    \mu_n(m^*) = \frac{\int_0^1 \mathrm{d}x \left[ \Phi_n(x; m^*) \right]^2}{\left[ \Phi_n(1; m^*) \right]^2},
\]

and will depend on the tip mass through \( m^* \). Note that if in equation (6) \( \mu_n \) is set to \( \mu_{\text{static}} = 0.2427 \) and \( n = 1 \), we recover the original Sader formula for \( k_{\text{static}} \) [19]. On the other hand, we would like to remark that calibration of the tipless rectangular cantilever’s first mode spring constant technically requires the application of equation (6) with \( n = 1 \) and \( \mu_1 = 0.25 \); however, the difference is clearly small.

In figure 1(b) the dependence of \( \mu_n \) for \( n = 1, 2 \) with respect to \( m^* \) is shown. As with the equivalent spring constant, the first eigenmode effective mass factor, \( \mu_1 \), tends to \( \mu_{\text{static}} \) in the limiting case of large \( m^* \). However, \( \mu_2 \) increases from 0.25 to 1 when \( m^* \) changes from 0 to 0.2. Consequently, equations (6) and (7) imply that the application of the original Sader formula to determine \( k_2 \) would entail an error of about 100% for a tip mass \( m_{\text{tip}} = 0.1m_{\text{beam}} \).

The correct application of equation (6) would involve an accurate knowledge of the eigenmode shape (equation (3)) or of the reduced mass, \( m^* \). Because this is rarely the case for most commercially available probes, we recommend use of the thermal noise method proposed by Hutter and Bechhoefer to calibrate spring constants of higher eigenmodes of non-rectangular and/or tipped cantilevers. This method provides the spring constant of a given eigenmode by measuring the temperature and thermal fluctuations under the desired resonance in the cantilever response power spectral density (PSD).

Most AFM experimental setups use a quadrature photodiode system [39], which measures the bending angle (slope) of the cantilever rather than its deflection or velocity. Thus, additional optical lever sensitivity calibration must be performed to relate the bending angle at the laser spot location to the deflection of the tip in each eigenmode. This can be performed either theoretically, with knowledge of the shape of the eigenmode [16, 36, 40], or experimentally [41]. However, in our experimental setup (see section 3), laser Doppler vibrometry was used to measure the cantilever’s velocity directly, which did not require the optical lever sensitivity calibration [42]. Consequently, in order to apply the thermal noise method to higher eigenmodes, the equipartition theorem had to be applied to the velocity, yielding the eigenmode effective mass instead of the spring constant itself [43]. Finally the latter could be calculated from mere multiplication by \( \omega_n^2 \).

\[
    k_n \text{thermal} = \frac{k_B T}{\langle q_n^2 \rangle} = \frac{\omega_n^2 k_B T}{\langle \dot{q}_n^2 \rangle}
\]

where \( q_n \) is the modal projection of the tip deflection, and where the symbols \( \langle \cdot \rangle \) and \( \dot{\cdot} \) denote, respectively, statistical average and temporal derivative of \( \cdot \). The Boltzmann constant is denoted by \( k_B \), while \( T \) represents the equilibrium temperature.

3. Methodology

To compare the Sader and thermal noise methods, thermal vibration time series were measured by laser Doppler vibrometry (Polytec MSA-400 Micro System Analyzer from Polytec GmbH, Waldbronn, Germany, laser spot size approximately 1 \( \mu \)m) at the free end of three different cantilevers: tipped rectangular, tipless rectangular and tipless picket, respectively. Their manufacturer, model, type, length and width are listed in table 1 (see figure 2 for scanning electron microscopy (SEM) images of the three cantilevers). For each cantilever, \( n_i \), velocity time series containing \( N \) points and sampled at a frequency \( f_s \) (see table 1) were measured. The PSD of the velocity time series was estimated using Welch’s periodogram method [44]. Each time series was divided into \( n_{\text{seg}} \) segments and the overall estimate of the PSD of each time series was taken as the average of the estimates from each segment. The sampling frequency \( f_s \) was selected based on each cantilever’s second natural frequency and the number of points was chosen such that the number of points inside
Table 1. Manufacturer specifications and PSD parameters for the AFM probes analyzed in this paper. \( L \) and \( b \) are, respectively, the cantilever length and width. For each cantilever, \( n \) time series containing \( N \) data points were sampled at a frequency \( f_s \). Each time series was then divided into \( n_{seg} \) segments and the PSD of each time series was estimated as the average of the PSD of each segment.

<table>
<thead>
<tr>
<th>Cantilever</th>
<th>Manufacturer</th>
<th>Model</th>
<th>Type</th>
<th>( L ) (( \mu )m)</th>
<th>( b ) (( \mu )m)</th>
<th>( n_{seg} )</th>
<th>( f_s ) (MHz)</th>
<th>( n_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mikromasch</td>
<td>NSC12 F</td>
<td>Tipped</td>
<td>250</td>
<td>35</td>
<td>2(^{11} )</td>
<td>128</td>
<td>1.024</td>
</tr>
<tr>
<td>2</td>
<td>Sandia</td>
<td>CADP</td>
<td>Tipless</td>
<td>300</td>
<td>29</td>
<td>2(^{10} )</td>
<td>128</td>
<td>0.512</td>
</tr>
<tr>
<td>3</td>
<td>AppNano</td>
<td>Forta TL</td>
<td>Picket</td>
<td>213</td>
<td>36</td>
<td>2(^{13} )</td>
<td>128</td>
<td>2.560</td>
</tr>
</tbody>
</table>

Figure 3(a) depicts the experimental PSD velocity thermal fluctuations measured at the free end of a tipped cantilever. The first two flexural resonances are marked by arrows and also enlarged in figures 3(b) and (c) respectively. Other peaks in the spectrum correspond to spurious resonances and electronic or mechanical noise. To estimate the thermal noise contained in the desired eigenmode resonance, the latter was normalized to its maximum, PSD\(_{\text{max}}\), and fitted to the PSD of a single harmonic oscillator (see figures 3(b) and (c)). The quality factor, \( Q_n \), and resonance frequency, \( f_n = \omega_n / 2\pi \), were obtained as fitting parameters. In order to avoid the influence from the noise floor, the eigenmode’s thermal fluctuations in velocity were determined as the integral of the fitted PSD,

\[
\langle \dot{q}_n^2 \rangle = \text{PSD}_{\text{max}} \int_0^\infty df \frac{(f/f_n)^2}{Q_n^2[1 - (f/f_n)^2]^2 + (f/f_n)^2}. \tag{9}
\]

Once the quality factor, resonance frequency and thermal fluctuations were obtained for each eigenmode, Sader and thermal noise methods were applied for both eigenmodes of all the three cantilevers. In particular, for the Sader method equation (6) was used with \( \mu_n = 0.25 \) and \( n = 1.2 \) respectively, yielding the first and second eigenmode equivalent spring constants corresponding to eigenmode shapes where \( m^* = 0 \). In that way, \( k_1 \) and \( k_2 \) were obtained from the original Sader method, rather than \( k_{\text{static}} \). Finally, to estimate \( k_n \) by means of the thermal noise method, equations (8) and (9) were used.

Sader’s method is frequently applied to a quality factor obtained from a driven/forced response (tuning curves), whereas in this experiment a quality factor obtained from a thermal response is used. These quality factors should theoretically be the same [19]. We have confirmed that these are similar by comparing the thermal and forced response for cantilever 3. An agreement in quality factor of 7% (not shown) was found between the two values.

A third method is provided as a reference to verify the experimental procedure and that the spring constants predicted from the thermal noise protocol are correct. This was achieved by using the scanning laser Doppler vibrometer to measure the PSD at multiple points along the \( x \)-axis of the cantilever (see insets in figure 3(a) for the normalized PSD profile along cantilever 1—tipped rectangular—at its first two eigenmodes respectively). Then, the experimental data were fitted to the square of the eigenmode given in equation (3) with \( n = 2 \), to directly determine the eigenvalue \( \alpha_2 \). From this and
Figure 3. (a) Velocity power spectral density (PSD) under thermal motion of cantilever 1 (tipped rectangular). The first two flexural resonances are marked by arrows and enlarged in (b) first eigenmode and (c) second eigenmode. The solid lines in (b) and (c) correspond to the fitting performed on each resonance to a single harmonic oscillator, with the eigenmode resonance frequency and quality factor as the only free parameters. The other two cantilevers yielded similar frequency responses (see tables 1 and 2 for specific details). The insets in figure 3(a) show the PSD profiles measured along cantilever 1’s $x$-axis, normalized to the value at the free end, for the first and second eigenmodes. (This figure is in colour only in the electronic version)

Table 2. First and second eigenmode resonance frequencies, $f_n$, quality factors, $Q_n$, and equivalent spring constants, calibrated by thermal noise, $k_n^\text{thermal}$, and Sader methods, $k_n^\text{Sader}$, for the cantilevers described in table 1. Error bars are calculated as the standard deviation of the estimates, obtained from the different time series taken for each cantilever. For the resonance frequency, error bars were calculated as the maximum between half of the PSD frequency resolution, $\Delta f = f_s/2N$, and the standard deviation.

<table>
<thead>
<tr>
<th>Cantilever</th>
<th>$f_1$ (kHz)</th>
<th>$Q_1$</th>
<th>$k_1^\text{thermal}$ (N m$^{-1}$)</th>
<th>$k_1^\text{Sader}$ (N m$^{-1}$)</th>
<th>$f_2$ (kHz)</th>
<th>$Q_2$</th>
<th>$k_2^\text{thermal}$ (N m$^{-1}$)</th>
<th>$k_2^\text{Sader}$ (N m$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>44.88 ± 0.03</td>
<td>150 ± 20</td>
<td>1.21 ± 0.04</td>
<td>1.2 ± 0.1</td>
<td>308.20 ± 0.08</td>
<td>76 ± 2</td>
<td>50 ± 5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>33.00 ± 0.06</td>
<td>82 ± 8</td>
<td>0.46 ± 0.02</td>
<td>0.43 ± 0.04</td>
<td>204.50 ± 0.06</td>
<td>22.6 ± 0.5</td>
<td>19 ± 2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>72.62 ± 0.02</td>
<td>139 ± 7</td>
<td>1.64 ± 0.04</td>
<td>1.63 ± 0.08</td>
<td>454.6 ± 0.1</td>
<td>420 ± 20</td>
<td>55 ± 2</td>
<td></td>
</tr>
</tbody>
</table>

The main results are summarized in table 2. Equivalent modal spring constants, $k_1$ and $k_2$, calculated by the thermal noise and Sader methods, as well as resonance frequencies and quality factors for the first and second eigenmodes, are shown for the three cantilevers. The corresponding value of each magnitude was calculated as the mean of the results obtained in the $n_t$ replicates, while the random error was characterized by the standard deviation. The differences between stiffnesses from Sader’s method and the thermal method reported in

4. Results and conclusions
The data in tables 2–4 demonstrate that there are important limitations to the Sader method when applied to higher eigenmodes due to their sensitivity to mass inhomogeneities at the free end, such as tip mass or picket geometry. On the other hand, the thermal noise method does not require knowledge of the eigenmode shape, and, as a result, is better suited for the calibration of higher eigenmodes. Furthermore, table 4 shows that mass inhomogeneities have a strong effect on the spring constant ratio $k_2/k_1$, a fact which may be important for probe selection in AFM methods which utilize the multiple eigenmodes \([47, 48]\).

In conclusion, we have compared the two most popular calibration methods, Sader and thermal noise methods, and their application to higher eigenmode spring constant determination in air, by means of laser Doppler vibrometry. Results were presented for AFM probes with two subtle, but common, mass inhomogeneities (tip mass and picket shape). We have found both protocols to agree within 7% in the first eigenmode, while the effect of mass inhomogeneities could lead to errors of up to 60% in the second eigenmode when using Sader’s method, due to deviations in the eigenmode shape with respect to tipless rectangular cantilever eigenmodes. Because the Sader method relies on geometric aspects and strictly considers tipless rectangular cantilevers, we propose the thermal noise method to be the method of choice when calibrating the spring constant of AFM probe higher eigenmodes.

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### Table 3. Thermal noise method versus Sader method spring constant calibration comparison for both eigenmodes, $n = 1, 2$. The first and third columns show the $k_{n,Sader}/k_{n,thermal}$ ratio from table 1 values. The second and fourth columns list the normalized deviation between the thermal noise and Sader methods, $R_n = \Delta k_{n,Sader}/\Delta k_{n,thermal}$, with $\Delta k_{n,Sader}$ representing the error bars for $k_{n,Sader}$ from table 1.

<table>
<thead>
<tr>
<th>Cantilever</th>
<th>$k_{1,thermal}/k_{1,Sader}$</th>
<th>$R_1$</th>
<th>$k_{2,thermal}/k_{2,Sader}$</th>
<th>$R_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.03 ± 0.09</td>
<td>0.1</td>
<td>1.5 ± 0.1</td>
<td>5.2</td>
</tr>
<tr>
<td>2</td>
<td>1.07 ± 0.07</td>
<td>0.7</td>
<td>1.2 ± 0.2</td>
<td>1.8</td>
</tr>
<tr>
<td>3</td>
<td>1.00 ± 0.04</td>
<td>0.1</td>
<td>0.78 ± 0.05</td>
<td>−4.0</td>
</tr>
</tbody>
</table>

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### Table 4. First and second eigenmode equivalent spring constant ratios, $k_2/k_1$, from thermal noise and Sader calibration methods, are compared with ratios predicted by equation (5), in two different ways. $k_2/k_1(m_{SEM}^*)$ is calculated by estimating the reduced tip mass, $m^*$, as the ratio of tip and cantilever volumes, from the SEM images in figure 2. $k_2/k_1(\Delta m_{SEM})$ is calculated by fitting measured second eigenmode shapes to equation (3), to determine the eigenvalue $\alpha_2$. Then, equation (5) is applied to calculate $k_2/k_1$. The brackets indicate upper and lower bounds of the corresponding estimate.

<table>
<thead>
<tr>
<th>Cantilever</th>
<th>$k_2/k_1_{thermal}$</th>
<th>$k_2/k_1_{Sader}$</th>
<th>$k_2/k_1_{fit}$</th>
<th>$k_2/k_1_{fit}$ $(m_{SEM}^*)$</th>
<th>$k_2/k_1_{fit}$ $(\Delta m_{SEM})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>62.8</td>
<td>41.7</td>
<td>57.8</td>
<td>(45.0, 65.8)</td>
<td>(0.02, 0.08)</td>
</tr>
<tr>
<td>2</td>
<td>49.1</td>
<td>44.2</td>
<td>43.8</td>
<td>39.0</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>33.5</td>
<td>43.6</td>
<td>32.4</td>
<td>27.7</td>
<td>−0.06</td>
</tr>
</tbody>
</table>
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