SELECTIVELY DOPED SEMICONDUCTOR HETEROSTRUCTURES

↓

HIGH MOBILITY TWO-DIMENSIONAL ELECTRON GAS (2DEG).

↓

INTEGER AND FRACTIONAL QUANTUM HALL EFFECTS (also WC).

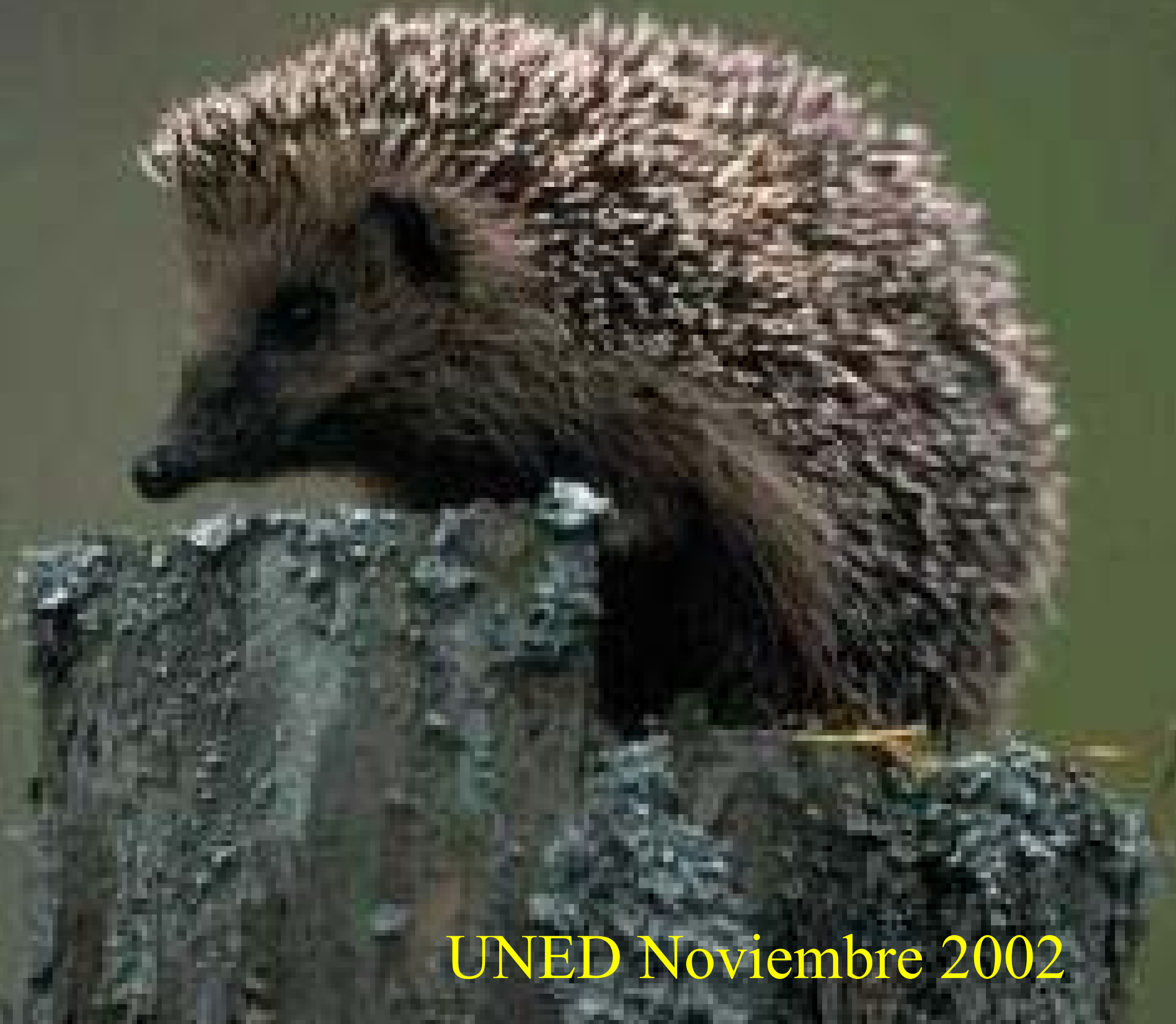
- **Fractional Statistics.**
- **Composite Bosons.**
- **Composite Fermions.**
- **Chiral Luttinger Liquids.**
- **Scaling theory of Localization.**
- ★ **Quantum Hall Ferromagnets.**
-

Quantum Hall Ferromagnets.

Luis Brey

CSIC

ICMM



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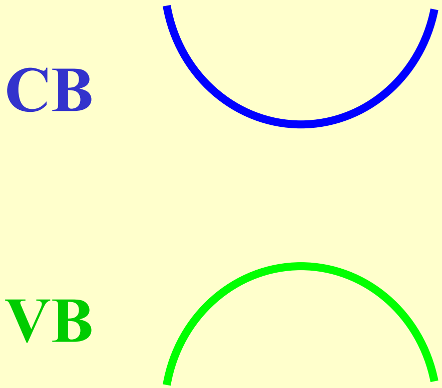
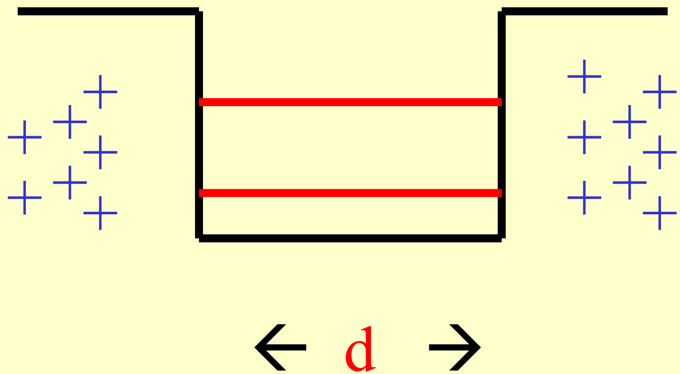
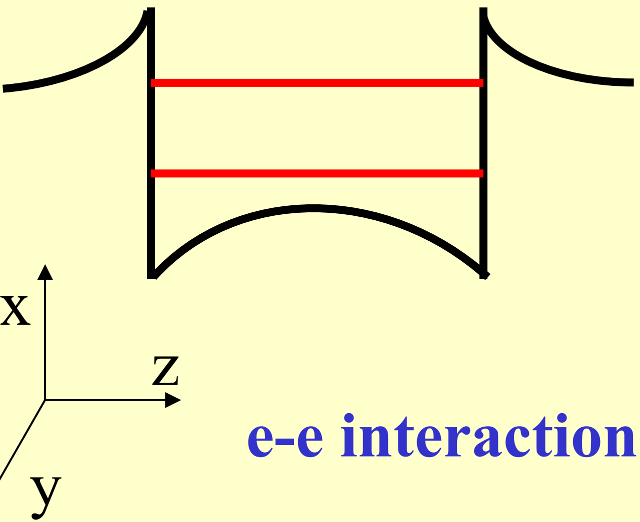
UNED Noviembre 2002

Two-Dimensional Electron Gas.



e^{ikr} ,m*, ε, g*





2DEG, free to move in x-y plane.

Confined in z-direction.

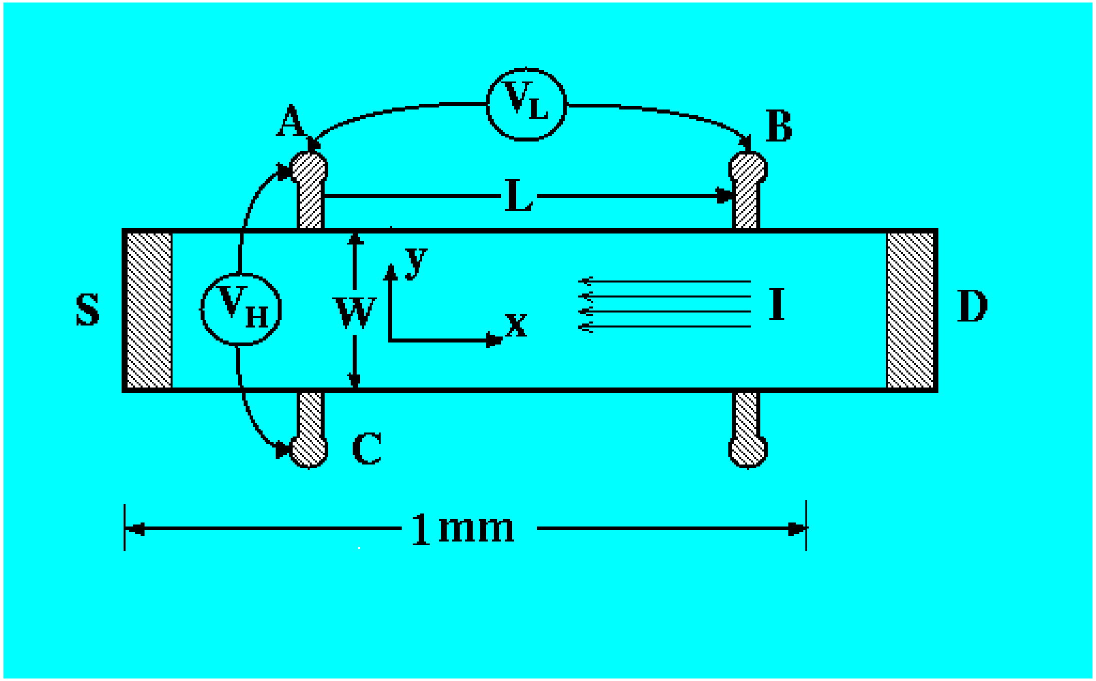
$E_2 - E_1 \gg kT, E_F$

$\lambda_F < d$

d=100Å, l=1000nm.

e-e interaction >> electron-impurity interaction.

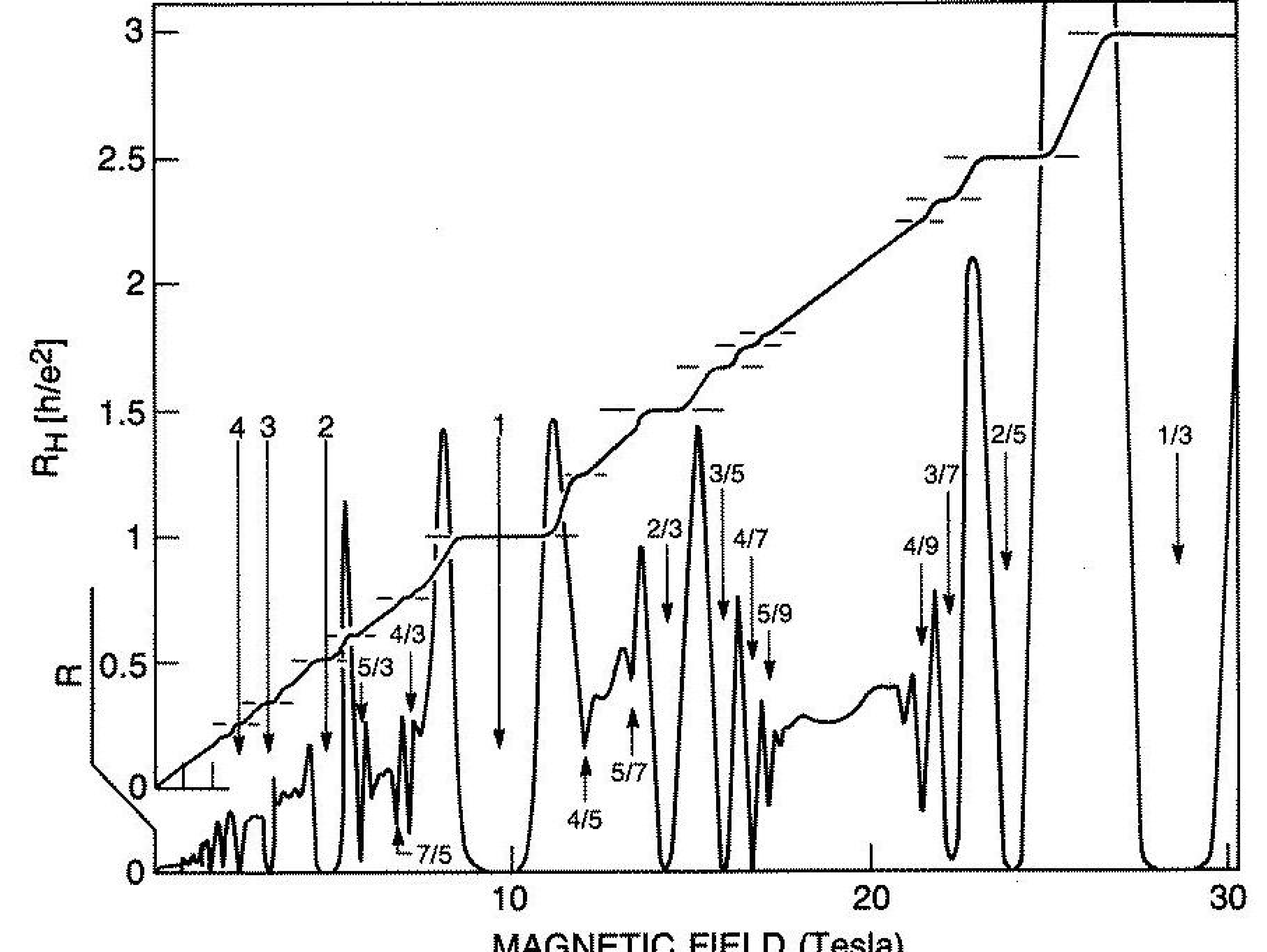
Quantum Hall Effects. 2DEG + B+ low T



$$R_H = \frac{V_H}{I} \qquad \frac{h}{e^2}\frac{1}{\nu}$$

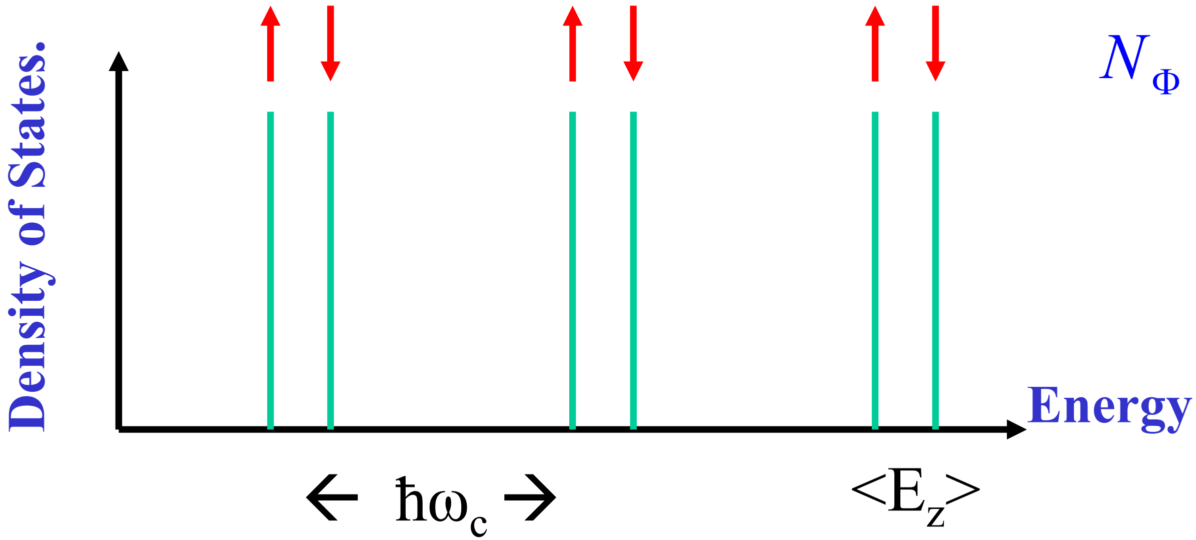
$$R_L = \frac{V_L}{I} \qquad 0$$

ν: integer IQHE.
fraction FQHE.



3
2.5
2
1.5
1
0.5
0
0
R_H [h/e^2]
R
4
3
2
1
4/3
5/3
7/5
4/5
5/7
2/3
3/5
4/7
5/9
4/9
3/7
2/5
1/3
10
20
30
MAGNETIC FIELD (Tesla)

Quantum Hall Ferromagnet. 2DEG in B.



$$N_\Phi = A\frac{B}{\Phi_0}, \quad \nu = \frac{N}{N_\Phi}$$

$$\ell \equiv \sqrt{\hbar c / eB}$$

$$\omega_c = \frac{eB}{m^* c}$$

ν: even integer. IQHE, gap is due to the quantization of the KE.
ν: fraction. FQHE, gap is due to electron-electron correlation
ν: odd integer. IQHE, gap is due to electron-electron interaction.

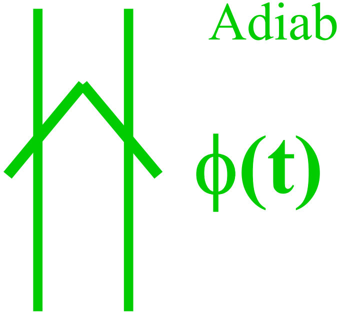
$E_z << KT$, $\quad e^2/\varepsilon\ell >> E_z$ **$2N_\Phi$places $N_\Phi$electrons**

Interactions are important for the spin-order.

Ground state is ferromagnetic even for $E_z \rightarrow 0$.

CHARGED EXCITATIONS. 2DEG at ν

Adiabatically.

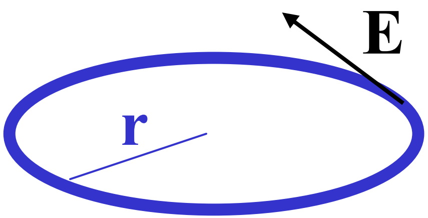


Faraday's law

$$2\pi r E = -\frac{1}{c}\frac{d\Phi}{dt}$$

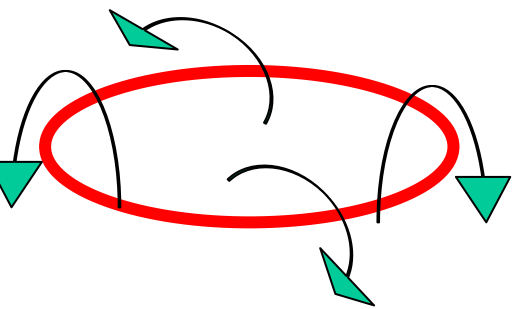
Hall effect.

$$J_t = \sigma_{xy} E$$



Continuity equation.

$$\frac{dQ}{dt} = -\sigma_{xy}\frac{1}{c}\frac{d\phi}{dt}$$



$$\Delta Q = -\nu e \frac{\Delta\Phi}{\Phi_0}$$

$$\Phi_0 = \frac{hc}{e}$$

$$\sigma_{xy} = \nu \frac{e^2}{h}$$

INTEGER QUANTUM HALL EFFECT $\nu=1$

- **Lowest Landau level occupied (Orbital and Spin).**
- **Ferromagnetic Ground State (even for g=0).** |↑↑↑- --↑↑↑↑---↑>
- **Spin Density Waves.** $4\pi\rho_s k^2$

  ρ_s : spin stiffness, loss of exchange Coulomb energy when spin orientation changes with position.

- **Charged excitations:**

  ↓

  add 1e to the system. |↑↑↑- --↑↑↑↑---↑>

  add 1 hole to the system. |↑↑↑- --↑_↑↑---↑>

- **Hartree Fock gap much bigger than the experimental gap.**

  Landau level mixing.
  Impurities.
  …..
  Charged excitation involve spin textures. (Rezayi, Shondi).

$\nu=1$

$|\uparrow\uparrow\uparrow---\uparrow\uparrow\uparrow\uparrow---\uparrow\rangle$

(Itinerant ferromagnet.)

Low energies, long distances

$$H = \frac{\rho_s}{2}\sum_{\mu}\int d^2r\left(\partial_{\mu}\vec{m}\right)^2 + \frac{E_z}{\pi\ell^2}\int d^2r\left(1+m_z\right)$$

ρ_s : spin stiffness. **E_z: Zeeman energy**

$\vec{m}$: unit vector field, parallel to the spin density.

$\vec{m}$ = (0,0,-1) (constant) Energy=0

NLσ model **NLO(3)**

-SDW

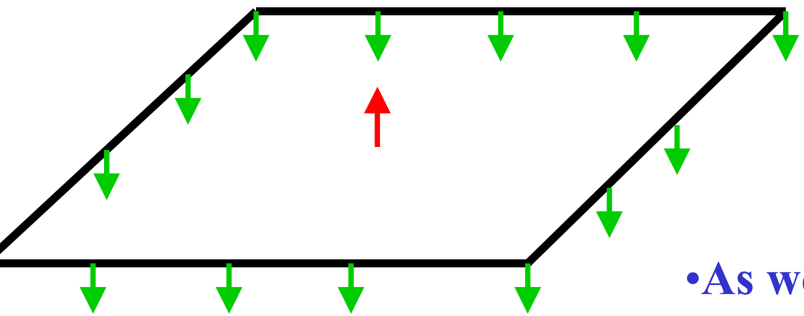
- SKYRMIONS.

WHAT IS A SKYRMION?

Finite energy solution of:

($\vec{m}$=cte, ferromagnetic GS, E=0)

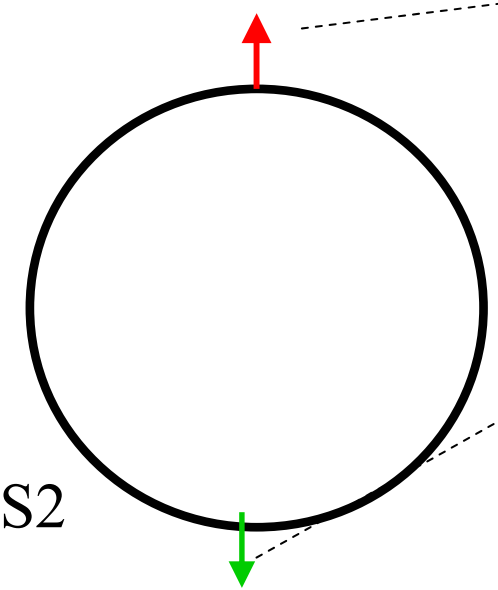
$$E = \frac{\rho_s}{2} \sum_{\mu} \int d^2 r \left(\partial_\mu \vec{m}\right)^2$$

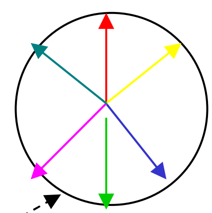


$$\lim_{|r| \to \infty} \vec{m} \to \vec{m}_0$$

- **As we move through the big sphere (real space), the vector field, *m*, moves in its unit sphere.**

Compact into a sphere

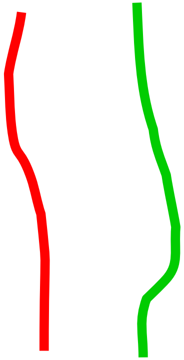




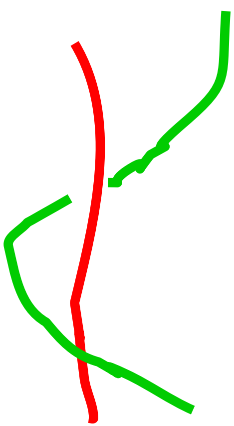
- **Solutions correspond to the mapping of one spherical surface S2 into another S2.**
- **All the solutions can be classified into homotopy sectors by INTEGERS: Q. (Number of times one of the spheres has been wrapped around the other. TOPOLOGICAL CHARGE.**

$$E = 4\pi\rho_s \,|\, Q \,|$$

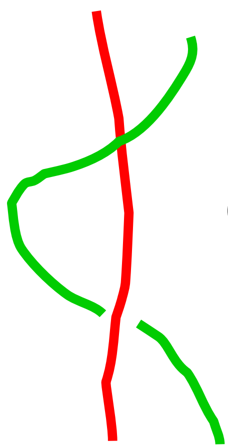
Topological charge.



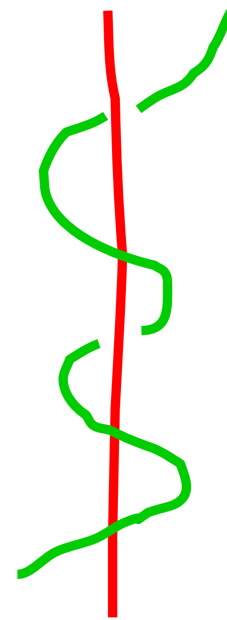
Q=0



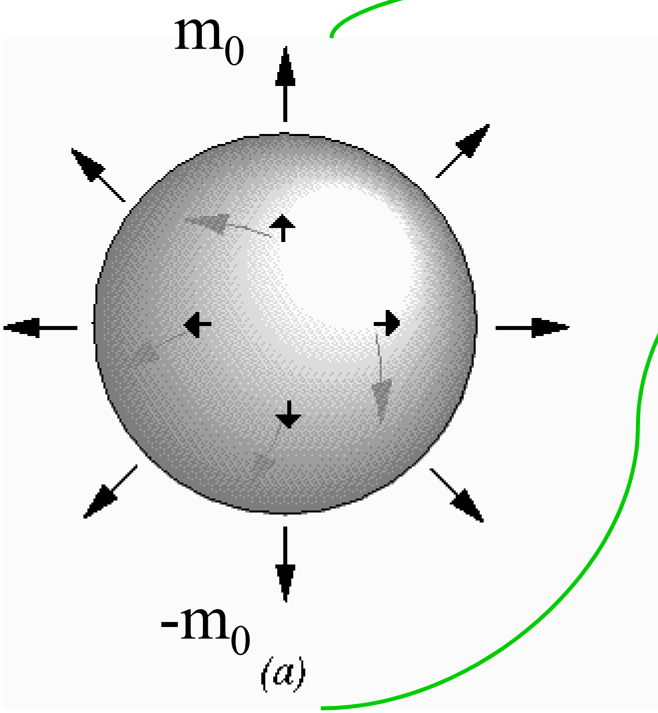
Q=1

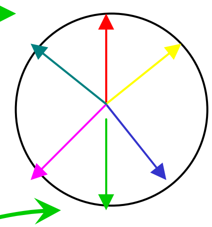


Q=-1



Q=2





Topological charge: The number of times one of the spheres has been wrapped around the othe

$$Q = \frac{1}{4\pi}\int dS^{unit} = \frac{1}{4\pi}\int d\Omega = \frac{1}{8\pi}\int \varepsilon_{\mu\nu}\,\vec{m}(\vec{r})\left[\partial_\mu \vec{m}(\vec{r}) \wedge \partial_\nu \vec{m}(\vec{r})\right] d\vec{r}$$

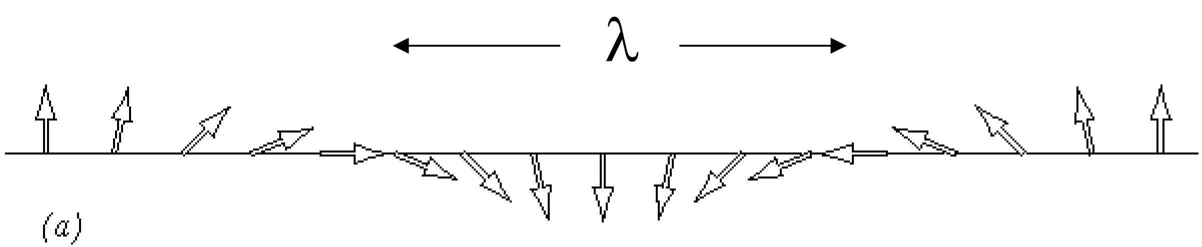
Pontryagian index density.

Jacobian of the change $d\Omega \rightarrow d^2r$

SKYRMION

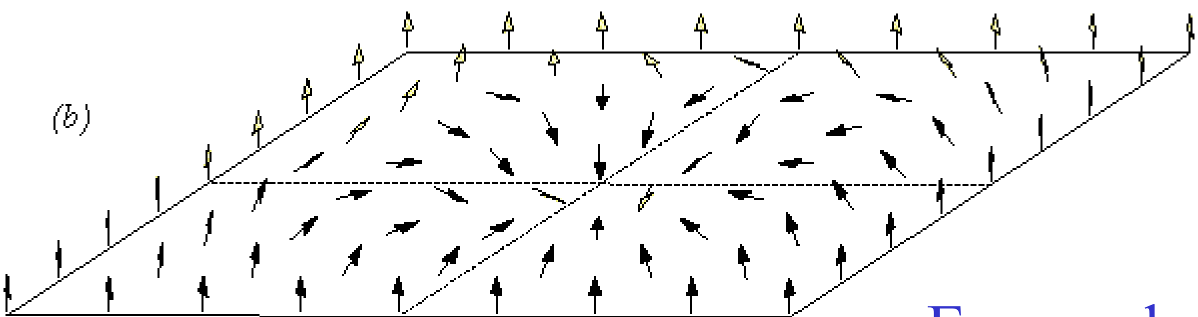
$$E = \frac{\rho_s}{2}\sum_{\mu}\int d^2r\left(\partial_\mu \vec{m}\right)^2$$

$$m_x = \frac{4\lambda x}{r^2+4\lambda^2}\ ,\quad m_y = \frac{4\lambda y}{r^2+4\lambda^2}\ ,\quad m_z = \frac{4\lambda^2 - r^2}{4\lambda^2+r^2}$$



$\vec{m}(0)=(0,0,-1)$

$\vec{m}(\infty)=(0,0,1)$

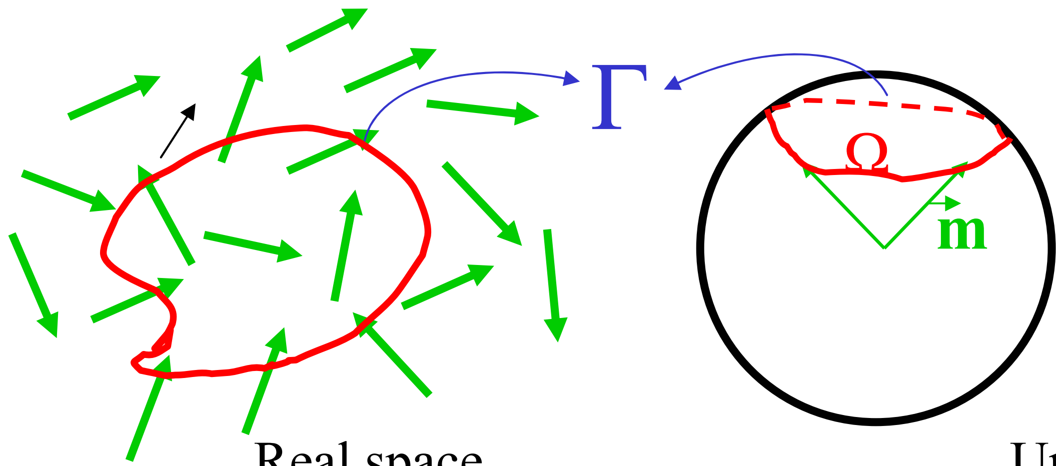


$$E = 4\pi\rho_s$$

Energy does not depend on λ.

NLσ model is scale invariant.

Why Skyrmions are important in the QHE regime?



Real space

Unit sphere

Berry's Phase $e^{i\Omega/2}$

Ω solid angle subtended by the arrow of the spin in its orbit.

$$\Omega = \int d\Omega = \frac{1}{2}\int \varepsilon_{\mu\nu}\,\vec{m}(\vec{r})\left[\partial_\mu \vec{m}(\vec{r}) \wedge \partial_\nu \vec{m}(\vec{r})\right] d\vec{r}$$

- This phase is exactly equivalent to having an Aharonov-Bohm phase due to additional magnetic flux $\Delta\Phi=\Phi_0\Omega/2$ inside the region limited by Γ.
- In the QHE regime, adding flux ⇔ adding electric charge.

$$\Delta Q = -e\frac{\Omega}{4\pi}\nu$$

Why Skyrmions are important in the QHE regime?

Due to incompressibility,

Topological charge =real charge

Charged excitation in the $\nu=1$ ferromagnet,

Skyrmions $$\overset{\qquad\qquad\qquad\downarrow}{|\uparrow\uparrow\uparrow\text{- --}\uparrow\uparrow\uparrow\uparrow\text{---}\uparrow\rangle}$$

Gap (Sk+AntiSK)=1/2Gap(e^-+h^+)

Sondhi ‘93

$S_{SK}=\infty$ $\quad$ $S=1/2$

(Zeeman=0.)

EFECTIVE HAMILTONIAN FOR DESCRIBE CHARGE EXCITATIONS AT ν=1.

$$H_{eff} = \frac{\rho_s}{2}\int d^2r\left(\vec{\nabla}\vec{m}\right) +$$

NLσ-model. Scale invariant.

$$+\frac{1}{2}\int d^2r\,\delta\rho(\vec{r})\,\frac{e^2}{\varepsilon\,|\vec{r}-\vec{r}\,'|}\,\delta\rho(\vec{r}\,')$$

Hartree term. O(3) symmetry
Large Skyrmions.

$$-\frac{1}{4\pi\ell^2}\,g^*\mu_B B\int\left(m_z(\vec{r})-1\right)d^2r$$

Zeeman Energy, privileges
z-direction. Small Skyrmions

Topological charge is equivalent to electric charge.

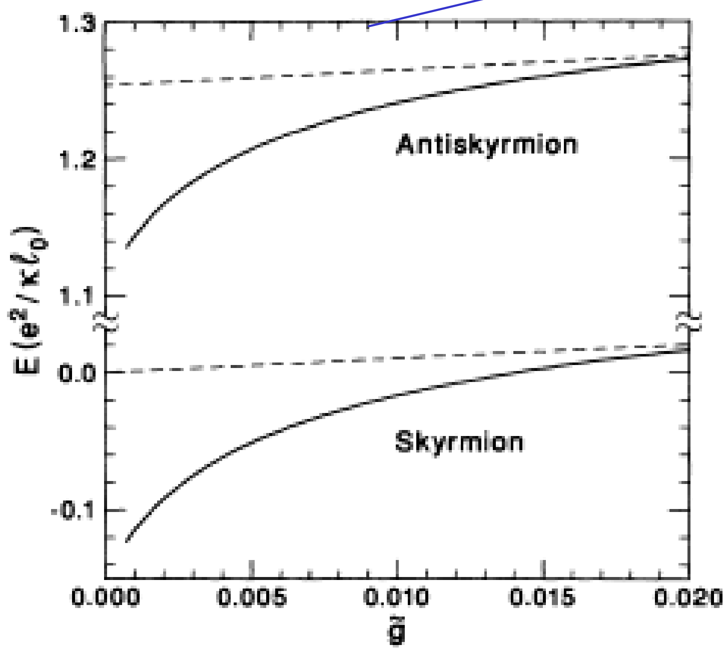
$$\delta\rho(\vec{r}) = -\frac{1}{4\pi}\;\varepsilon_{\mu\nu}\vec{m}(\vec{r})\left[\partial_\mu\vec{m}(\vec{r})\wedge\partial_\nu\vec{m}(\vec{r})\right]$$

PHYSICAL REVIEW B VOLUME 50, NUMBER 15 15 OCTOBER 1994-I

Charged spin-texture excitations and the Hartree-Fock approximation in the quantum Hall effect

↓

|↑↑↑- --↑↑↑↑---↑>



$g\mu_B B \sim 0.05 e^2/\varepsilon\ell$

$S_{SK} \sim 3.5$

VOLUME 74, NUMBER 25 PHYSICAL REVIEW LETTERS 19 JUNE 1995

Optically Pumped NMR Evidence for Finite-Size Skyrmions in GaAs Quantum Wells near Landau Level Filling $\nu = 1$

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VOLUME 75, NUMBER 23 PHYSICAL REVIEW LETTERS 4 DECEMBER 1995

Evidence for Skyrmions and Single Spin Flips in the Integer Quantized Hall Effect

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VOLUME 76, NUMBER 4 PHYSICAL REVIEW LETTERS 22 JANUARY 1996

Evidence of Skyrmion Excitations about $\nu = 1$ in n-Modulation-Doped Single Quantum Wells by Interband Optical Transmission

E. H. Aifer and B. B. Goldberg
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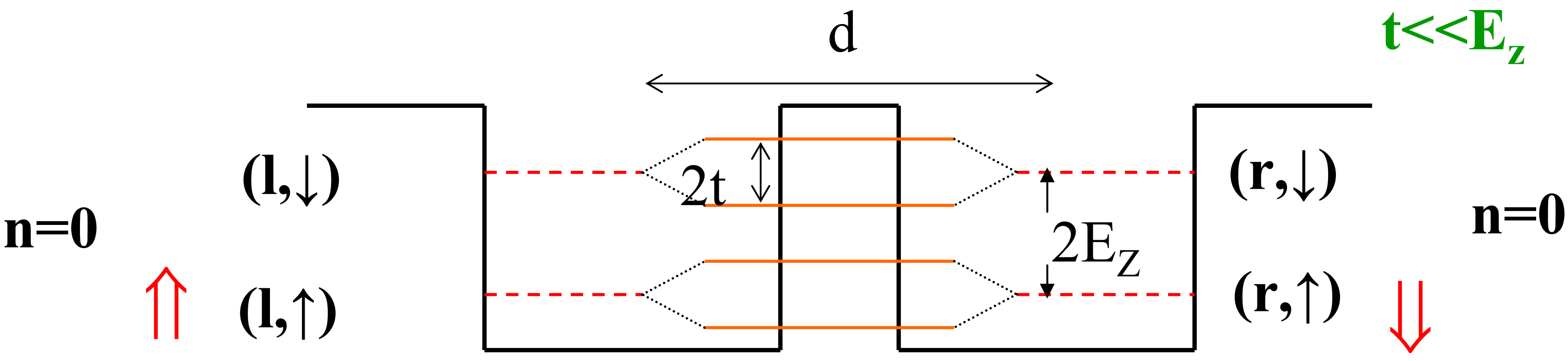
D. A. Broido
Physics Department, Boston College, Chestnut Hill, Massachusetts 02167

OTHER QUANTUM HALL FERROMAGNETS.

QH EASY PLANE (XY) FERROMAGNETS.

DQW system at ν=1

$t \ll E_z$



- ISO-MAGNETISM.
- Capacitive energy $\langle S_z \rangle = 0$.
- Isospin in the xy plane.

EASY PLANE FERROMAGNET.

Linear Goldstone mode.

Bimerons.

QH-ISING FERROMAGNETS (1).

$\nu = 2$

n=0,↓ n=1,↑

$\hbar\omega_c$ vs. E_Z electrons

$2N_\Phi$ states and N_Φ electrons.

n=0,↑ (inert)

Hamiltonian

$$H = \sum \left[\hbar\omega_c(n+\tfrac{1}{2}) + \sigma E_z\right] C^+_{nk\sigma} C_{nk\sigma} +$$
$$\sum V_{n,m,n',m'}(k',k,q)\, C^+_{nk+q\sigma} C^+_{n'k'-q\sigma'} C_{m'k'\sigma'} C_{mk\sigma}$$

Spin-Orbit Coupling

$$H_{so} = \lambda_1 \sum (\sigma^x p_y - \sigma^y p_x) + \lambda_2 \sum (\sigma^x p_x - \sigma^y p_y)$$

$\lambda_2 \rangle \lambda_1$ **and**

$$H_{so} = \lambda \sum \left(C^+_{n=1,\uparrow,X} C_{n=0,\downarrow,X} + h.c. \right)$$

QH-ISING FERROMAGNETS (2).

$\Downarrow$ n=0,↓ $\Uparrow$ n=1,↑

n=0,↑ (inert)

m(r)= <σ (r)>

m=(0,0,1) **Ferromagnetic stat**

m=(0,0,-1) **Unpolarized state.**

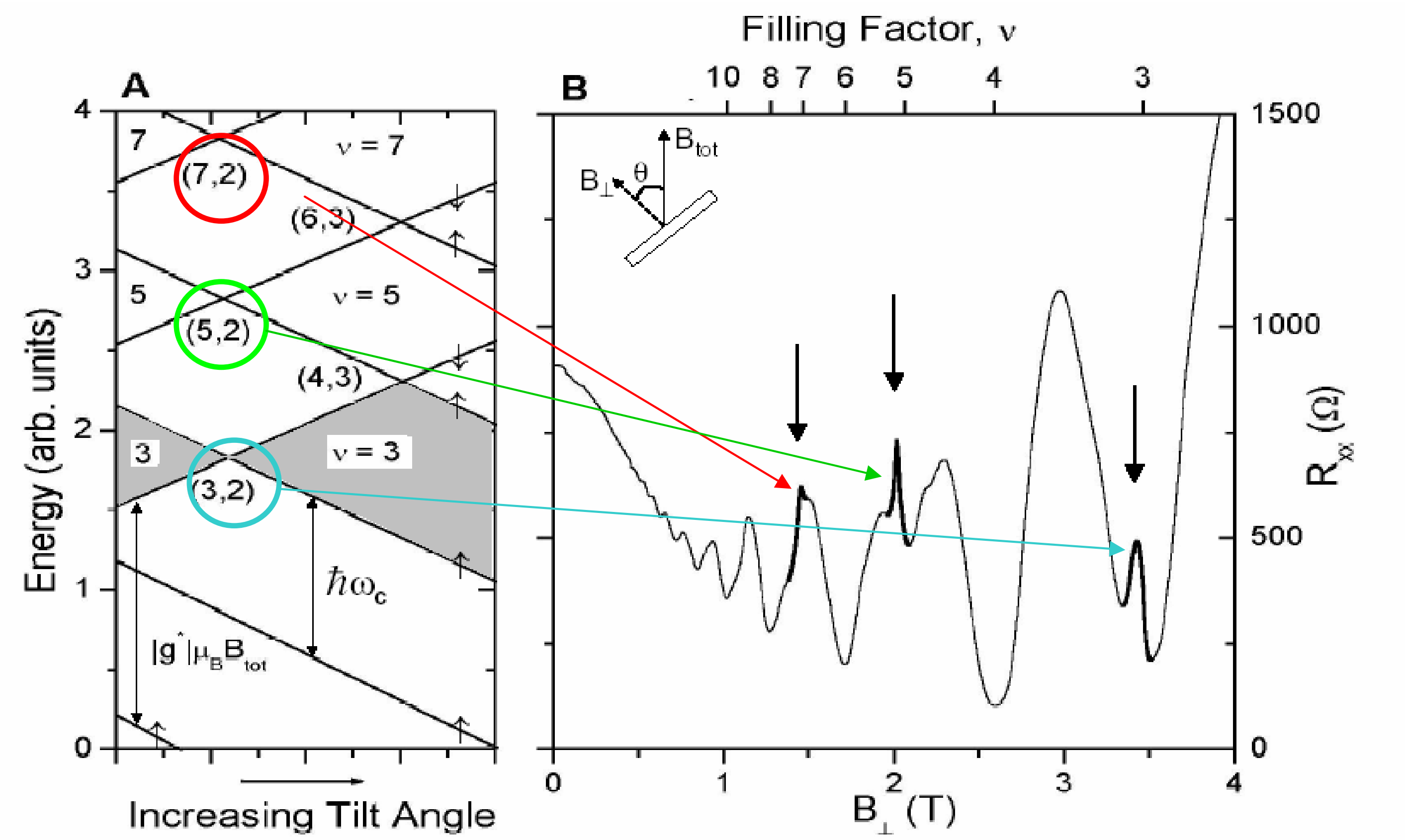
Energy functional for isospin textures, F(m(r)).

$$F_{2D} = \alpha \int d\vec{r}\, m_z(\vec{r}) + \beta \int d\vec{r}\, m_z^2(\vec{r}) + \frac{\rho_{par.}}{2} \int d\vec{r} \left[\partial_\mu m_z(\vec{r})\right]^2 + \frac{\rho_{perp}}{2} \int d\vec{r} \left[\partial_\mu \vec{m}_{perp}(\vec{r})\right]^2 + \tilde{\rho} \int d\vec{r} \left[\partial_\mu^2 \vec{m}_{perp}(\vec{r})\right]^2 + \lambda \int d\vec{r}\, m_x(\vec{r}) + V$$

Coefficients α,β, and ρ's are related with EZ, ħωc, selfenergies and stiffness in the IQHE regime.
Transversal stiffness ρ_{perp} <0 ; n and n' different parity, possible helical phase. ρ controls the patial variation of m_{perp}.
Topological and electric charge are equivalents , Hartree term.

β<0 1st order

QH-ISING FERROMAGNETS (3).

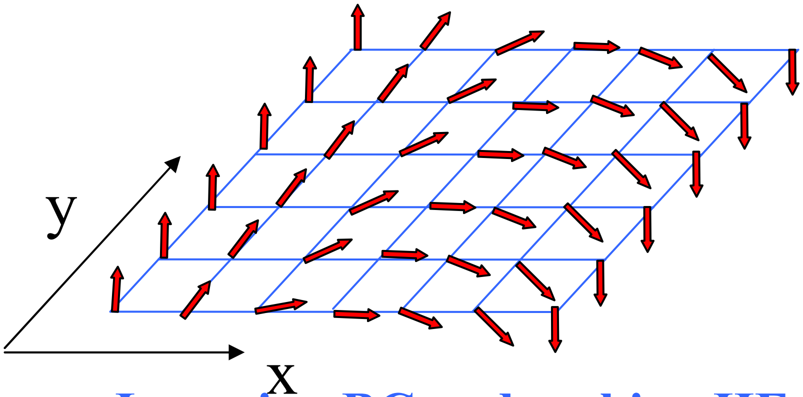


Resistance Spikes at Transitions Between Quantum Hall Ferromagnets.

Poortere et al. Science (2000)

DOMAIN WALLS IN QH ISING F. (1).

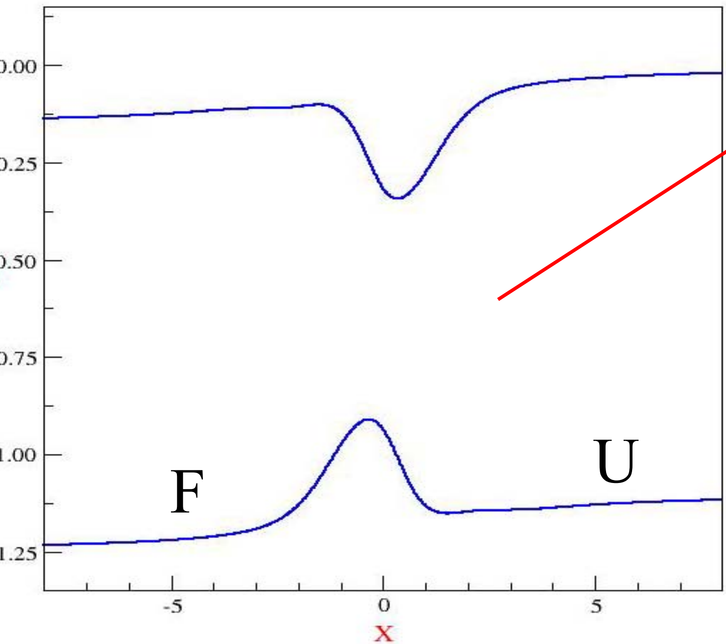
$\alpha=0$, U and F states are degenerated. Disorder and T create DW's.



$m_z=+(-)1$ at $x=-(+)\infty$

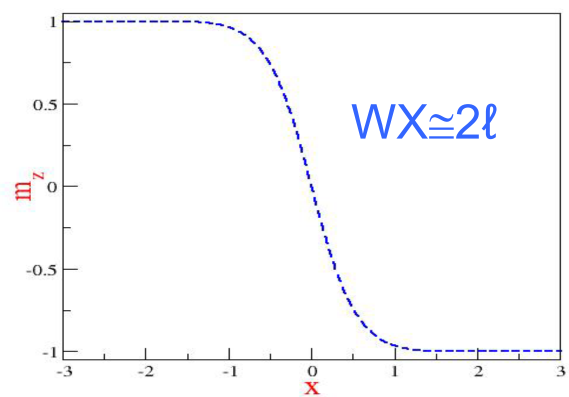
$$m_z=\cos 2\theta(x)$$
$$m_x=\sin 2\theta(x)$$

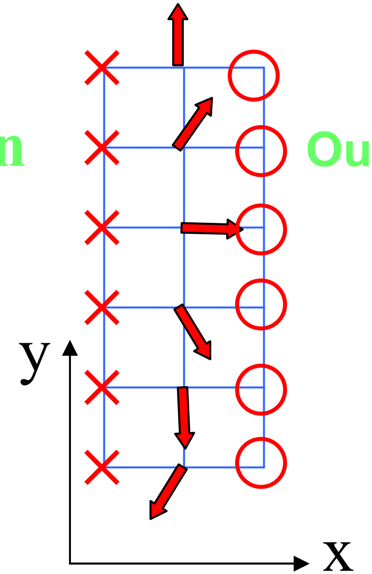
Imposing BC and making HF calculations,we get the structure of the DW.



The reduction of the energy gap at the DW is an indication of the loss of coherence of the wavefunction at the DW.

Energy density, $\gamma\ell \cong 0.045 e2/\varepsilon\ell$





DOMAIN WALLS IN QH ISING F. (2).

We have neglected rotations of $\mathbf{m}_{perp}$.

$$m_z = \cos 2\theta(x)$$
$$\vec{m}_{perp} = \sin 2\theta(x) e^{i\Phi(y)}$$

By integrating the 2D functional and performing HF calculations,

$$F_{DW}[\Phi(y)] = \frac{\rho}{2}\int dy[\partial_y \Phi(y)]^2 + \beta \int dy \left[(\partial_y \Phi(y))^4 + (\partial_y^2 \Phi(y))^2\right] + V_H - \lambda_{so}\int dy \cos\Phi(y)$$

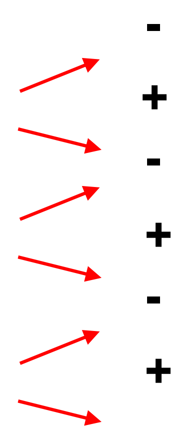
•$\rho<0$. β controls the spatial variation of $\Phi(y)$.

•Solutions are characterized by $Q = \int \Phi(y) dy / (2\pi)$.

Q=0

•ρ, B and λ_{so}. A fan phase is expected, but the Hartree energy prevents this isospin texture.

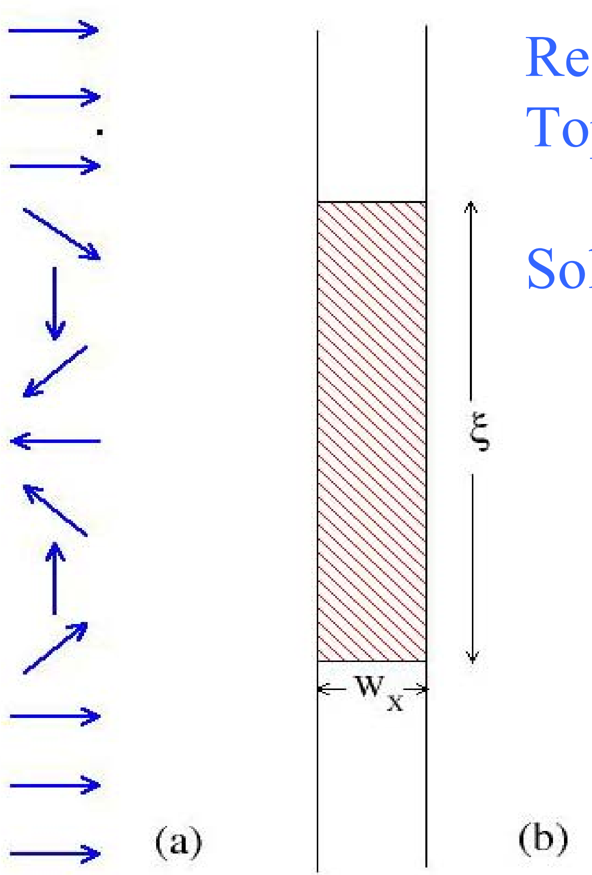
•SO coupling $\rightarrow$ isospin pointing in +x direction.



CHARGED DOMAIN WALL (1).

Q=1.

$\Phi(y)$ rotates from 0 to 2π when going from $-\infty$ to ∞ (soliton)



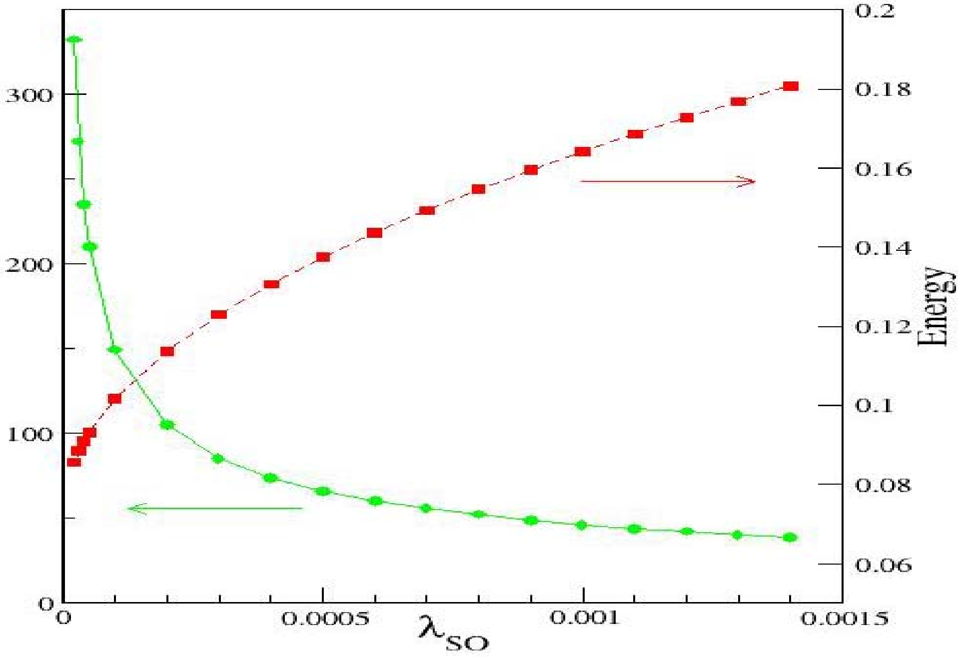
Relevant charged excitations.
Topological charge is equivalent to electric charge.

Solitons ≈ Skyrmions trapped in the DW.

Stiffness $\rho<0$. Small solitons. $\xi\downarrow$.
Spin-Orbit coupling. Small solitons. $\xi\downarrow$.
Hartree Energy. Large solitons. $\xi\uparrow$.

CHARGED DOMAIN WALL (2).

Energy and size of the soliton at ν=2.



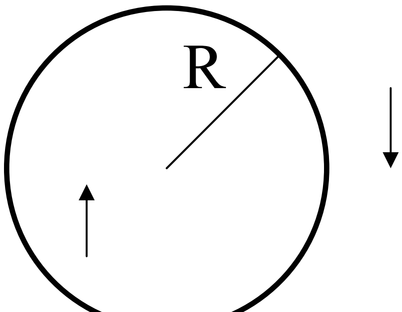
•λ_{SO} (GaAs) ~2x10^{-4} to 8x10^{-4}

Energy≤0.15$e^2/\varepsilon\ell$<< DW HF gap ~1 e^2/ε

•Solitons can be the relevant charged excitations in QH-Ising-F (with DW's).

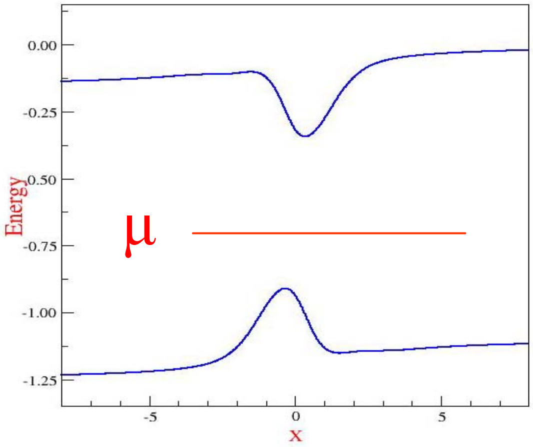
•**In absence of DW, a carrier can create a domain, and get located at the DW.**

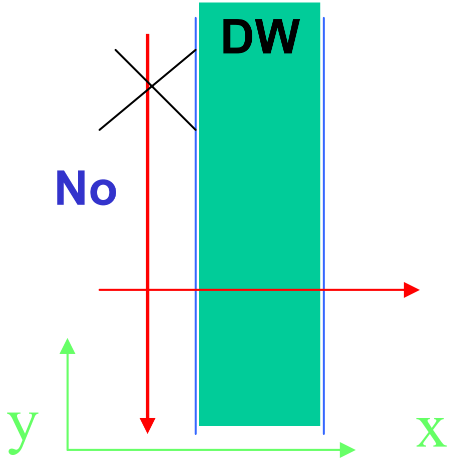
$$2\pi R\gamma + E_{soliton} < E_{QP}^{HF}$$



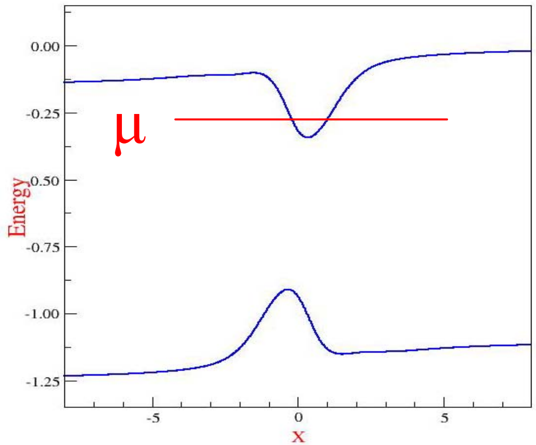
TRANSPORT PROPERTIES OF THE DW (1).

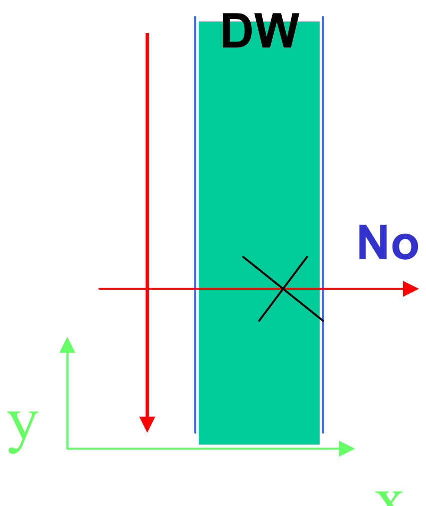
is located at the gap of the DW. No current can flow paralell to the DW. The carriers pass across the DW.





μ is in a band, perfect transition along the well. No current trough the well.

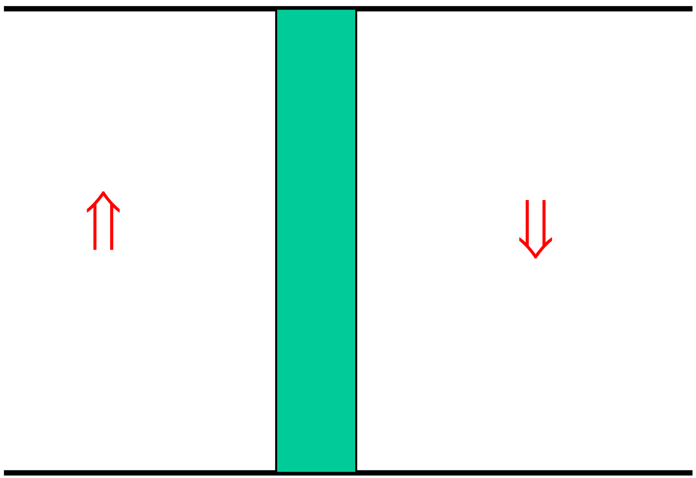




TRANSPORT PROPERTIES OF THE DW (2).

- The same arguments apply for the charged excitation gap. Soliton energy.
- The current through a DW separating a F form the U phase is $\neq 0$ only if μ lies in a charged excitation gap of the DW.

- If $\lambda_{SO}=0$ then $\xi \to \infty$. No gap for charged soliton. No transport across the DW.
- If $\lambda_{SO} \neq 0$. Gap for charged soliton. Charge can cross the DW.



Spin conservation

Resistance spikes are related with the transport across the DW.

<u>CONCLUSIONES.</u>

La interacción entre electrones tiene efectos dramáticos, en el orden de spin
n sistemas Hall cuánticos..
El estado fundamental a ciertos valores de llenado es ferromagnetico. La
arga eléctrica es equivalente a la topológica. Las excitaciones cargadas son
kyrmions.
DQW son ferromagnetos de plano fácil.
La condición de coincidencia produce ferromagnetos de eje fácil.

Hemos estudiado DW’s.

Los portadores se atrapan en los DW y forman solitones.

La energía y tamaño de los solitones están determinadas por el acoplo s-o.