

SPINS, CHARGES and CURRENTS

at

DOMAIN WALLS

in a

QUANTUM HALL ISING FERROMAGNET.

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OUTLINE

- **Magnetism in QHE systems.**
Isotropic, easy-plane and easy-axis.
- **QH Ising Ferromagnets.**
- **Energy functional.**
- **Spin-Orbit Coupling.**
- **Domain Walls.**
- **Charged Domain Walls.**
- **Transport Properties of DW's.**

MAGNETISM in QHE SYSTEMS.

- **2DEG + B** Eigenstates are highly degenerated

$$N_{\phi} = SB/\phi_0 .$$

And this degeneracy is responsible for a variety of new phenomena as FQHE, Wigner Cryst.,

QUANTUM HALL FERROMAGNETISM

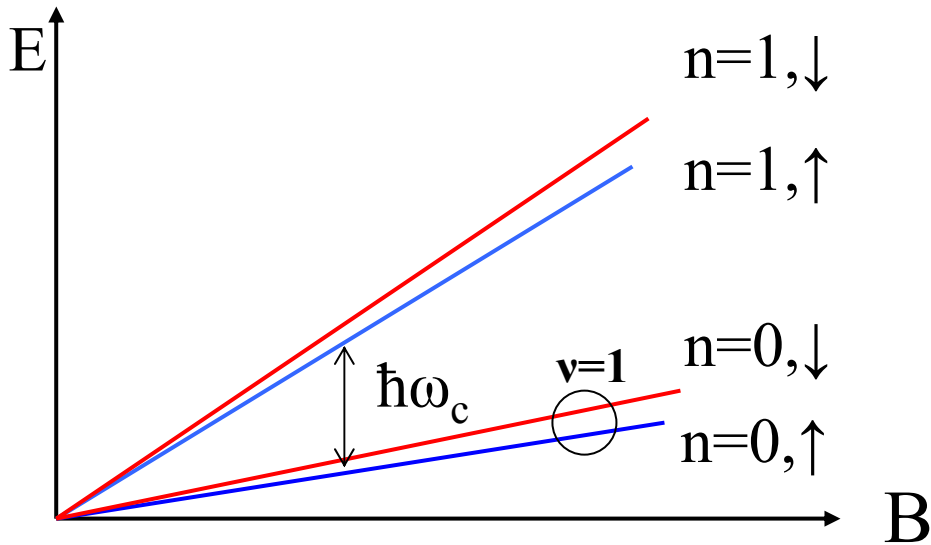
- Magnetism in the QHE regime has the unique property that

Topological and electric charges
are equivalents.

Low energy Collective Excitations (SW).

Topological Excitations.

QH ISOTROPIC FERROMAGNETS.



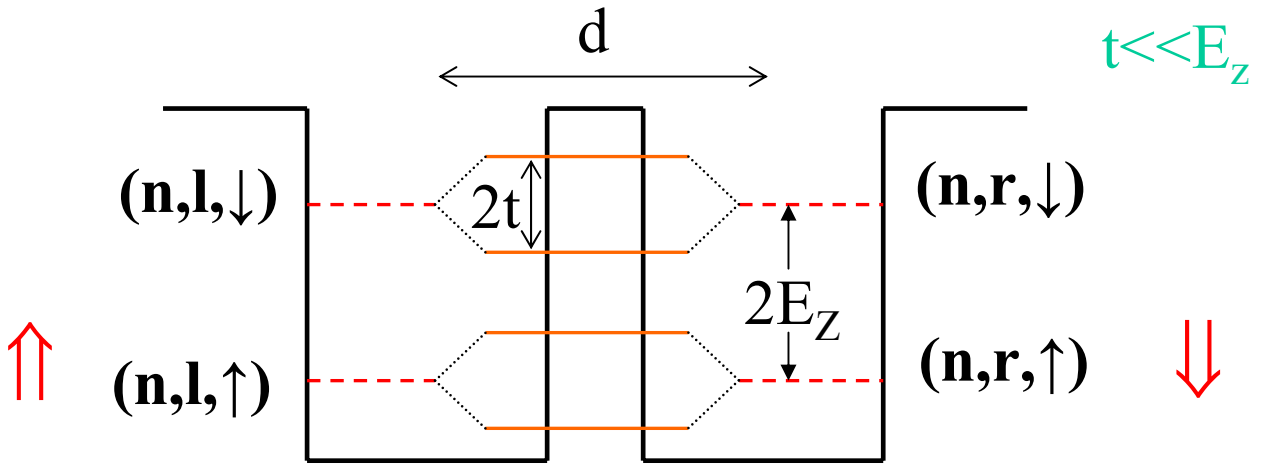
- At odd integer QHE, $\nu=2n+1$, when the states (n, \uparrow) and (n, \downarrow) are degenerated (close) in energy.
- There are $2N_\Phi$ degenerated states and N_Φ electrons.
- The electron spins can point in any direction, but exchange interaction provides a ferromagnetic coupling.

Isotropic Ferromagnet -NLO(3).

- Spin waves.
- Topological excitations, SKYRMIONS.

QH EASY PLANE (XY) FERROMAGNETS.

DQW system at $\nu=4n+1$ (3)



- **ISO-MAGNETISM.**
- **Capacitive energy $\langle S_z \rangle = 0$.**
- **Isospin in the xy plane.**

EASY PLANE FERROMAGNET.

Linear Goldstone mode.
Bimerons.

QH-ISING FERROMAGNETS.

- Occurs when the states (n', \uparrow) and (n, \downarrow) cross in energy, at integer filling factor. $E_z \sim \hbar\omega_c$
- Due to the different form of the Landau level wavefunctions the transition is discontinuous.

Easy Axis Ferromagnet.

Playing with subband index and Landau level index it is possible to change the system from easy plane to easy axis.

T.Jungwirth et al. PRL'98

In DQW with large d and n bigger than zero, stripes phases can be stabilized.

- Brey and Fertig PRB 2000.
- Demler et al. 2001.
- Pan et al. PRB (2001).

QH-ISING FERROMAGNETS (1).

v=2

(CF 2/3)

n=0, ↓

n=1, ↑

$\hbar\omega_c$ vs. E_z electrons

$e^2/\epsilon l$ vs E_z CF's.

n=0, ↑ (innert)

$2N_\Phi$ states and N_Φ electrons.

Hamiltonian.

$$H = \sum \left[\hbar\omega_c \left(n + \frac{1}{2} \right) + \sigma E_z \right] C_{nk\sigma}^+ C_{nk\sigma} + \sum V_{n,m,n',m'}(k',k,q) C_{nk+q\sigma}^+ C_{n'k'-q\sigma}^+ C_{m'k'\sigma'} C_{mk\sigma}$$

Spin-Orbit Coupling

$$H_{SO} = \lambda_1 \sum (\sigma^x p_y - \sigma^y p_x) + \lambda_2 \sum (\sigma^x p_x - \sigma^y p_y)$$

$\lambda_2 \gg \lambda_1$ and

$$H_{SO} = \lambda \sum \left(C_{n=1,\uparrow,X}^+ C_{n=0,\downarrow,X} + h.c. \right)$$

QH-ISING FERROMAGNETS (2).

$$\begin{array}{ll} n=1, \uparrow & \uparrow \\ n=0, \downarrow & \downarrow \end{array} \quad \mathbf{m}(\mathbf{r}) = \langle \boldsymbol{\sigma}(\mathbf{r}) \rangle$$

$\mathbf{m}=(0,0,1)$ **Ferromagnetic state**
 $\mathbf{m}=(0,0,-1)$ **Unpolarized state.**

Energy functional for isospin textures, $F(\mathbf{m}(\mathbf{r}))$.

$$\Psi = \prod_X \left[\cos \theta(X) C_{X\uparrow}^+ + \sin \theta(X) e^{i\psi(X)} C_{X-G,\downarrow}^+ \right] |0\rangle$$

$|0\rangle$ is the vacuum, all the $n=0, \uparrow$ states occupied.

Assuming that θ and ψ change slowly and G is small,

$$m_z(\mathbf{x}) = \cos 2\theta(\mathbf{x})$$

$$m_x(\mathbf{x}, y) + im_y(\mathbf{x}, y) = \sin 2\theta(\mathbf{x}) e^{i(\psi(\mathbf{x}) + Gy)}$$

QH-ISING FERROMAGNETS (3).

$$\begin{aligned} F_{2D} = & \alpha \int d\vec{r} m_z(\vec{r}) + \beta \int d\vec{r} m_z^2(\vec{r}) \\ & + \frac{\rho_{par.}}{2} \int d\vec{r} \left[\partial_\mu m_z(\vec{r}) \right]^2 + \frac{\rho_{perp.}}{2} \int d\vec{r} \left[\partial_\mu \vec{m}_{perp}(\vec{r}) \right]^2 \\ & + \hat{\rho} \int d\vec{r} \left[\partial_\mu^2 \vec{m}_{perp}(\vec{r}) \right]^2 + \lambda \int d\vec{r} m_x(\vec{r}) + V_H \end{aligned}$$

- Coefficients α, β , and ρ 's are related with E_Z , $\hbar\omega_c$ and selfenergies and stiffness in the IQHE regime.
- Transversal stiffness $\rho_{perp} < 0$; n and n' different parity, possible helical phase. ρ controls the spatial variation of \mathbf{m}_{perp} .
- Topological and electric charge are equivalents,

$$q(\vec{r}) = \frac{\epsilon_{\mu,\nu}}{8\pi} \vec{m} \cdot \left(\partial_\mu \vec{m} \times \partial_\nu \vec{m} \right)$$

it is necessary to include the Hartree term.

QH-ISING FERROMAGNETS (4).

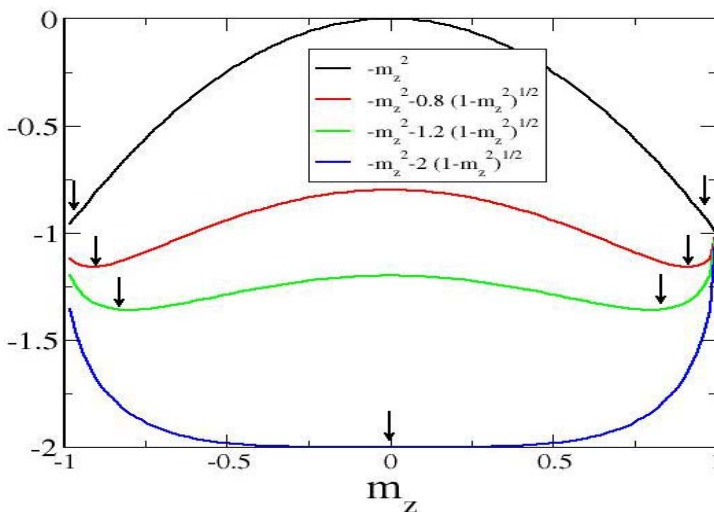
$$\begin{aligned}
 F_{2D} = & \alpha \int d\vec{r} m_z(\vec{r}) + \beta \int d\vec{r} m_z^2(\vec{r}) \\
 & + \frac{\rho_{par.}}{2} \int d\vec{r} \left[\partial_\mu m_z(\vec{r}) \right]^2 + \frac{\rho_{perp.}}{2} \int d\vec{r} \left[\partial_\mu \vec{m}_{perp}(\vec{r}) \right]^2 \\
 & + \hat{\rho} \int d\vec{r} \left[\partial_\mu^2 \vec{m}_{perp}(\vec{r}) \right]^2 + \lambda \int d\vec{r} m_x(\vec{r}) + V_H
 \end{aligned}$$

- Phase transition between Ferromagnetic ($n=1, \uparrow$) and Unpolarized ($n=0, \downarrow$) phases occurs when,

$$\alpha = \frac{1}{8\pi} (\Sigma_{0000} - \Sigma_{1111} - \Sigma_{1010}) - \left(E_z - \frac{\hbar\omega_c}{2} \right) \frac{1}{2\pi} = 0.$$

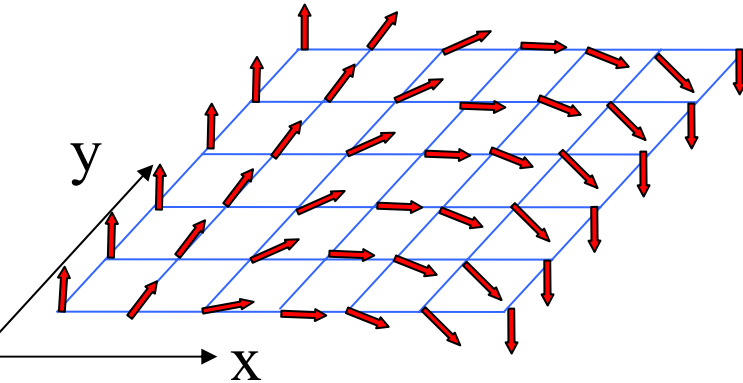
$$E_z=0, \hbar\omega_c=0.47e^2/\epsilon l$$

- Order of the transition depends on β, ρ 's y λ ,
- $\beta < 0$ 1st order.
- Large ρ_{perp} , helical phase, 2nd order. $\left\{ \begin{array}{l} v=2. \text{ No big enough.} \\ v=2/3 ?? \end{array} \right.$
- Large λ implies 2nd order. $\left\{ \begin{array}{l} \text{No for electrons} \\ \text{Possible for holes.} \end{array} \right.$



DOMAIN WALLS IN QH ISING F. (1).

$\alpha=0$, **U** and **F** states are degenerated.
Disorder and **T** create **DW**.

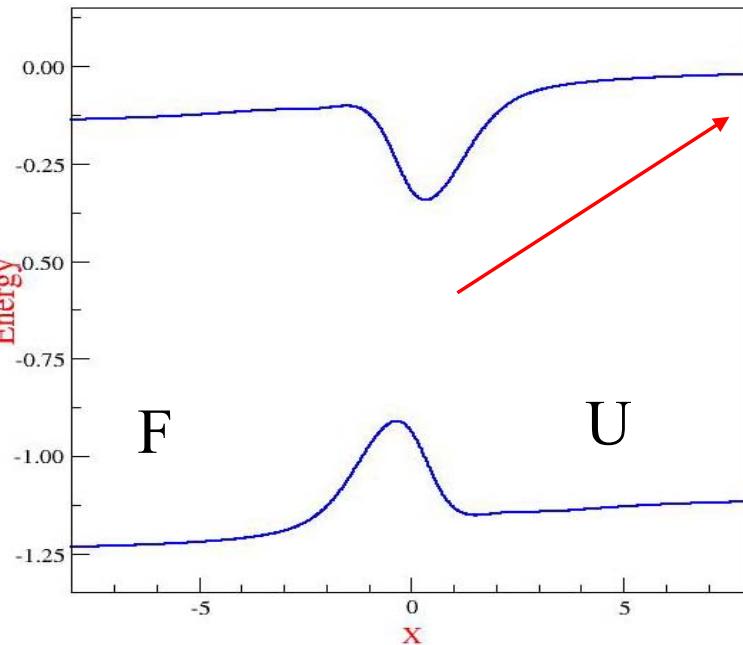


$$m_z = +(-)1 \text{ at } x = -(+)\infty$$

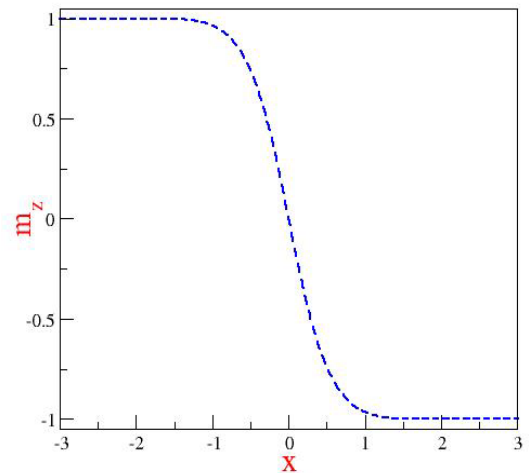
$$m_z = \cos 2\theta(x)$$

$$m_x = \sin 2\theta(x)$$

Imposing BC and performing HF calculations,
we obtain the electronic structure of the DW.



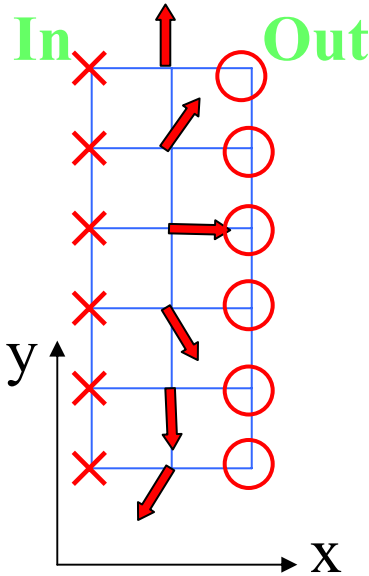
The reduction of the energy gap at the DW is an indication of the loss of coherence of the wavefunction at the DW.



$$W_x \approx 2\ell$$

$$\text{Energy density, } \gamma\ell \approx 0.045e^2/\varepsilon\ell$$

DOMAIN WALLS IN QH ISING F. (2).



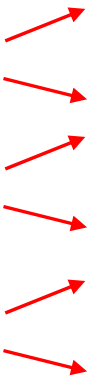
We have neglected rotations of \mathbf{m}_{perp} .

$$\begin{aligned} \mathbf{m}_z &= \cos 2\theta(\mathbf{x}) \\ \vec{\mathbf{m}}_{\text{perp}} &= \sin 2\theta(\mathbf{x}) e^{i\Phi(\mathbf{y})} \end{aligned}$$

By integrating the 2D functional and performing HF calculations,

$$F_{DW}[\Phi(\mathbf{y})] = \frac{\rho}{2} \int d\mathbf{y} [\partial_y \Phi(\mathbf{y})]^2 + B \int d\mathbf{y} \left[(\partial_y \Phi(\mathbf{y}))^4 + (\partial_y^2 \Phi(\mathbf{y}))^2 \right] + V_H - \lambda_{so} \int d\mathbf{y} \cos \Phi(\mathbf{y})$$

- $\rho < 0$. B controls the spatial variation of $\Phi(\mathbf{y})$.
- Solutions are characterized by $Q = \int \Phi(\mathbf{y}) d\mathbf{y} / (2\pi)$.
- $Q=0$.



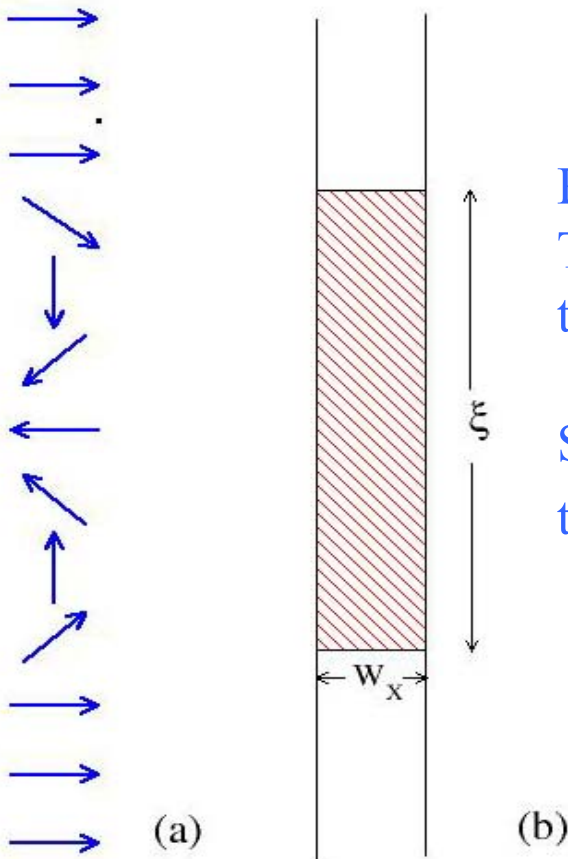
• ρ , B and λ_{so} a fan phase is expected, but the Hartree energy prevent this isospin texture.

• SO coupling \rightarrow isospin pointing in $+x$ direction.

CHARGED DOMAIN WALL (1).

Q=1.

$\Phi(y)$ rotates from 0 to 2π when going from $-\infty$ to ∞ (soliton).



Relevant charged excitations.
Topological charge is equivalent to electric charge.

Solitons \approx Skyrmions
trapped in the DW.

Stiffness $\rho < 0$. Small solitons.

$\xi \downarrow$.

Spin-Orbit coupling. Small solitons.

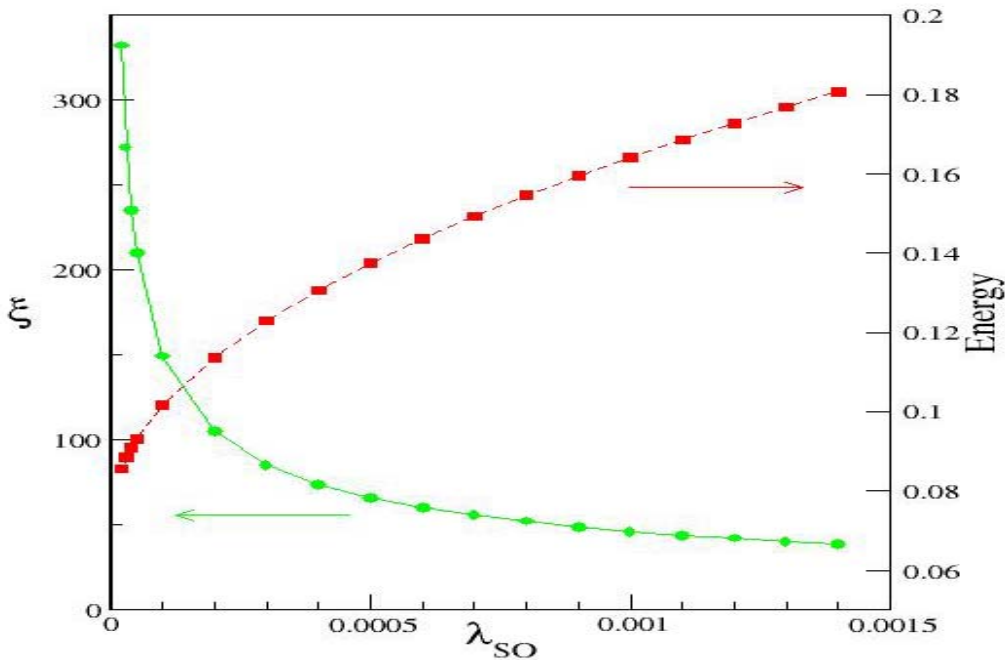
$\xi \downarrow$.

Hartree Energy. Large solitons.

$\xi \uparrow$.

CHARGED DOMAIN WALL (2).

Energy and size of the soliton at $\nu=2$.



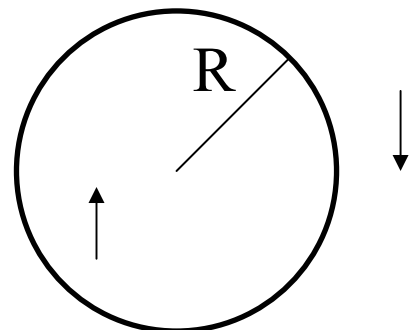
• λ_{SO} (GaAs) $\sim 2 \times 10^{-4}$ to 2×10^{-5} ,

Energy $\leq 0.15 e^2 / \epsilon \ell \ll$ DW Hartree-Fock gap $\sim 1 e^2 / \epsilon \ell$.

• Solitons can be the relevant charged excitations in QH-Ising-F in presence of DW.

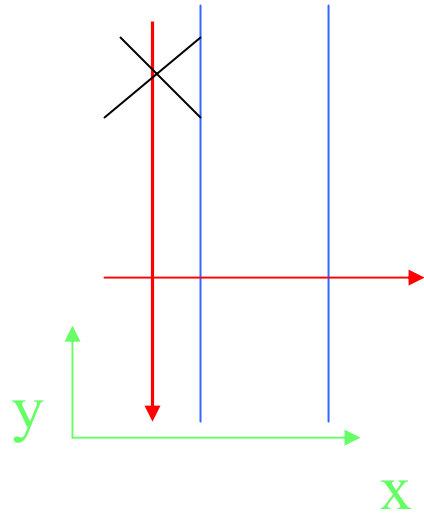
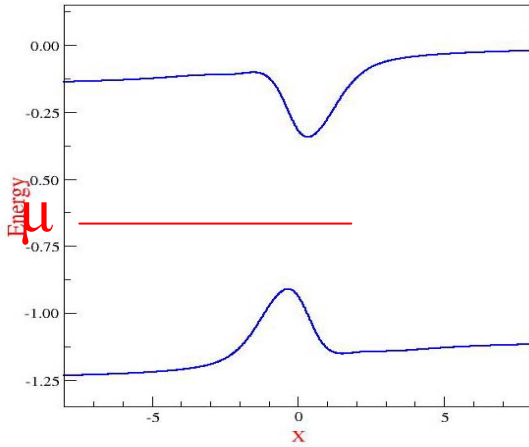
• In absence of DW, a carrier can create a domain, and get located at the DW.

$$2\pi R\gamma + E_{soliton} < E_{QP}^{HF}$$

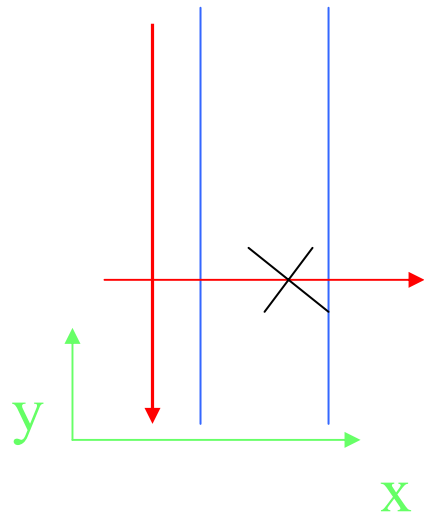
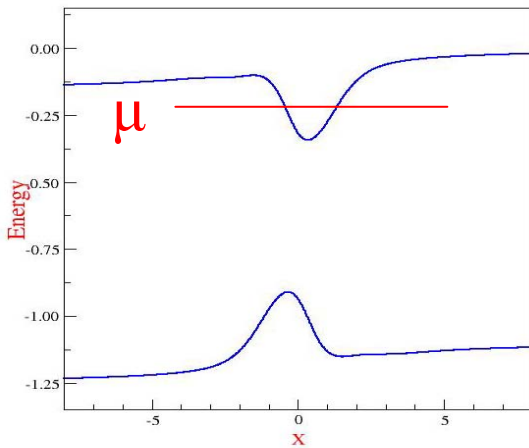


TRANSPORT PROPERTIES OF THE DW (1).

μ is located at the gap of the DW. No current can flow parallel to the DW.
The carriers pass across the DW.



μ is in a band, perfect transition along the well. No current through the well.



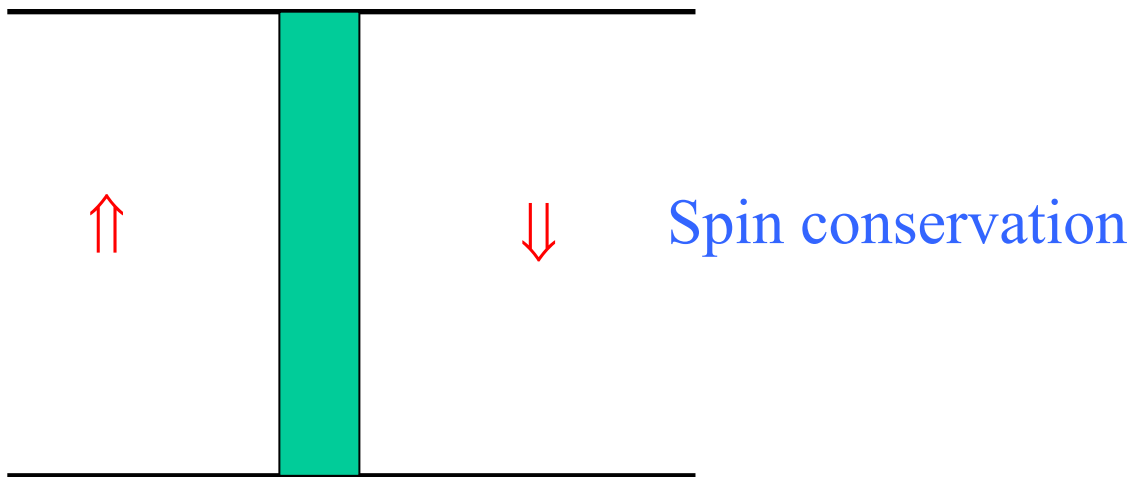
• The same arguments apply for the charged excitation gap.
Soliton energy.

• The current through a DW separating a F from the U phase is $\neq 0$ only if μ lies in a charged excitation gap of the DW.

TRANSPORT THROUGH A DW (2).

If $\lambda_{SO}=0$ then $\xi \rightarrow \infty$. No gap for charged soliton.
No transport across the DW.

If $\lambda_{SO} \neq 0$. Gap for charged soliton.
Charge can cross the DW.



Small charged excitation energy. Weak coupling between states of the two domains.
Small current with large resistance.

SUMMARY.

Functional energy for QH-Ising-Ferromagnet.

- Negative stiffness.

Electrical properties of DW's.

- Neutral DW's. Hartree term is important.
- Carriers get trapped in DW's and form solitons.
- Energy and size of the solitons are determined by Spin-Orbit coupling.
- Transport properties of DW's.